# Incremental Coding over MIMO Channels

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***Abstract*—The problem of multicasting common data to several users over multiple-input multiple-output (MIMO) Gaussian channels is studied. A closed-loop setup is considered where the channel matrices are known to the transmitter and respective receivers. An incremental-redundancy (rateless) scenario is considered, where the effective rate is measured by the time that each user needs to stay online until it is able to decode the message. A practical transmission scheme for the two-user case is proposed which, by linear pre- and post-processing combined with successive decoding and interference cancellation, transforms the two MIMO channels into a set of parallel channels with no loss of mutual information, where each user needs to tune in for a duration of time proportional to its individual capacity. This scheme is used for designing a practical transmission scheme for the Gaussian MIMO half-duplex relay channel. We then turn to the related scenario of transmission to a single user over a MIMO channel with unknown but constant signal-to-noise ratio (SNR), for which we develop an optimal low-complexity hybrid ARQ coding scheme, which is optimal for two SNRs and propose a scheme for more SNRs, the loss of which vanishes when the SNRs are high. Finally, we show that even when applied to single-input single-output ("scalar") channels, the scheme provides a practical solution for cases not covered by previous work.**

## I. Introduction

Incremental redundancy (IR) (or "rateless") coding plays an important role in many communication problems as a means to efficiently cope with channel uncertainty. The design of IR coding schemes recently has received renewed strong interest in the coding community, motivated by a number of emerging applications in Gaussian networks, e.g., as a means of facilitating relaying and multicasting.

A remarkable example of such codes for *erasure* channels are the Raptor codes of Shokrollahi [1], which build on the LT codes of Luby [2], [3]. An erasure channel model (for packets) is most appropriate for rateless coding architectures anchored at the application layer, where there is little or no access to the physical layer. Apart from erasure channels, there is a growing interest in exploiting rateless codes closer to the physical layer, where AWGN models are more natural; see, e.g., [4] and the references therein. Accordingly, considerable work has been done on designing good (capacity-approaching) IR codes for the (scalar) AWGN channel, see e.g., [4]–[9]. Incorporating multiple antennas at both the transmitter and the receiver, may allow great improvement over their single-antenna counterparts. Thus, rateless transmission over multiple-input multiple-output (MIMO) channels looks appealing and has received recent attention, see, e.g., [10]–[12].

In this work we consider the case of multicasting the same message to several users over different Gaussian MIMO channels ("broadcast channel") sharing the same input, where we assume perfect knowledge of all channel matrices ("closed loop") at all transmission ends. We focus on the case of two users, for which we derive a transmission scheme that by linear pre- and post- processing transforms the problem into a set of virtual parallel AWGN channels, over which standard (fixed-rate) codes designed for the scalar AWGN channel may be used.

In "classical" multicasting, one wishes to optimize the input probability distribution such that the minimal mutual information (MI) (corresponding to the worst channel w.r.t. this input) is maximized. This way, one guarantees this same transmission rate to all the users, whereas users which have higher MI do not exploit their excess rates, and wait until the end of the transmission block before recovering the message. In some scenarios, however, this excess MI can be utilized to shorten the transmission period required by the stronger users to recover reliably the message, by designing an appropriate rateless scheme. This is useful in different relaying scenarios; see, e.g., [13]–[15] and references therein.

From an information-theoretic prospective, constructing such a scheme is possible [16]. From a coding perspective, however, such a scheme requires the design of special codes, as well as joint encoding and decoding of all antennas signals, which is computationally demanding.

An interesting special case of the aforementioned problem is that of a single user transmitting over a known channel but unknown signal-to-noise ratio (SNR), both remaining constant ("slow fading") throughout the whole transmission.

In this work we derive an optimal, yet low-complexity, rateless scheme for transmitting a common message to two users over MIMO channels. The approach builds on a recent work [17] which presents a coding scheme for *equal-rate* transmission of a common message to two users. We extend the latter scheme by combining space and time dimensions to obtain effective higher-dimensional MIMO channels for the augmented inputs and outputs, and demonstrate that this scheme becomes useful even when designing rateless codes for single-input single output (SISO) channels.

This approach is then applied for the Gaussian MIMO relay channel (depicted in Figure 1), when working in "half-duplex" mode. In the "half-duplex" mode, the relay cannot receive and transmit at the same time, and is restricted to performing only

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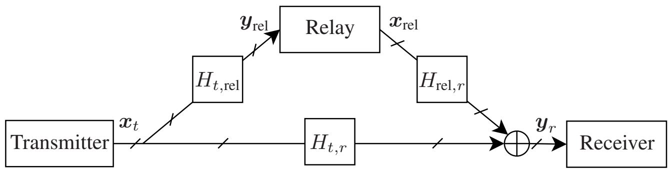


Fig. 1. Gaussian MIMO relay channel.

one of these tasks at each time instance.

Mitran et al. [13] proposed a practical scheme for this scenario, which employs rateless coding: Transmission is divided into two phases, where during the first ("listening") phase, the transmitter multicasts to the relay and the receiver, at the end of which the relay is able to recover the message conveyed by the transmitter; during the second ("collaboration") phase, both the relay and the transmitter transmit *coherently*, until the receiver is able to decode the message as well.[1] In [13], knowledge of the channel matrices at the respective receivers (destination nodes) only was assumed, and not at the transmitters (originating nodes). Thus, restricting attention to white channel inputs only was sufficient. Note that such a scheme requires the use of codes for channels with varying SNR, which are not easy to construct and require a "bit-loading" mechanism which impairs designing a *practical* scheme that approaches the optimum performance of this setting. Since in our problem we assume perfect knowledge of all channel statistics everywhere, this allows to further improve this scheme in terms of achievable rates as well as approaching it using linear transformations, successive interference cancellation (SIC) and *fixed-rate* scalar Gaussian channel codes.

In the current work we show how this optimum performance can be achieved using linear transformations along with successive interference cancellation (SIC) and good scalar *fixed-rate* AWGN codes.

We further discuss the problem of transmission over a known channel with unknown SNR, where the SNR can take one of several distinct values. For the case of two possible SNRs (corresponding to two blocklengths), we use the proposed two-user MIMO rateless scheme, whereas for more SNRs we show how using the universal channel decomposition [18], along with scalar rateless codes, allows to approach optimality up to a loss which vanishes for high SNRs.

In this work we concentrate mainly on the case of two users. Nonetheless, extensions to more than two users can be constructed, as explained in [19].

## II. The MIMO Broadcast Rateless Problem

We consider a channel model where the received signal of user $i$ ($i = 1, 2$) is given by

$$\boldsymbol{y}_i = H_i \boldsymbol{x} + \boldsymbol{z}_i \,, \tag{1}$$

where $\boldsymbol{y}_i$ is a $N_r^{(i)} \times 1$ vector, $\boldsymbol{x}$ denotes the $N_t \times 1$ complex-valued input vector limited to an average power $P$ per symbol, $H_i$ is the $N_r^{(i)} \times N_t$ complex channel matrix to user $i$, and $\boldsymbol{z}_i$ is assumed to be a circularly-symmetric Gaussian vector of zero mean and identity covariance matrix (We denote matrices by capital letters. Vectors are denoted by bold letters).

The rate achievable with an $N_t \times N_r$ channel matrix $H$ and input covariance matrix $K \triangleq E\{\boldsymbol{x}\boldsymbol{x}^H\}$ is equal to the Gaussian MI between the input and output vectors:

$$R(H, K) \triangleq \log \left| I_{N_r} + H K H^\dagger \right| \,, \tag{2}$$

where $|\cdot|$ denotes the determinant and $I_{N_r}$ is an identity matrix of dimension $N_r$. The point-to-point capacity of user $i$ is given by $C_i = R(H_i, K_i)$, where $K_i$ is the covariance matrix maximizing (2) for $H = H_i$.

In a two-user rateless setting, the transmitter needs to send the same $k$ bits to both receivers. Each user "listens" to the transmission from time instance $1$ until it is able to reliably decode all bits, then it may tune out. The *online time* of user $i$, $n_i$, is the number of channel uses which it requires, and the resulting effective rates are given by:

$$R_i = \left\lfloor \frac{k}{n_i} \right\rfloor , \quad i = 1, 2 \,, \tag{3}$$

where $\lfloor \cdot \rfloor$ denotes the "floor" operation. The following, due to Shulman [16]. states the optimal rates.

*Proposition 1:* The effective rate pair $(R_1, R_2)$ is achievable under power constraint $P$ if and only if there exists a covariance matrix $K$ with trace$\{K\} \leq P$ such that:

$$\frac{R(H_i, K)}{R_i} + C_i \left[ \frac{1}{R_i} - \frac{1}{R_{\bar{i}}} \right]^+ \geq 1 \,, \qquad i = 1, 2 \,,$$

where $[a]^+ \triangleq \max\{a, 0\}$ and

$$\bar{i} = \begin{cases} 2 & i = 1 \\ 1 & i = 2 \end{cases} .$$

This result can be understood as follows. Without loss of generality, assume that $R(H_1, K) \geq R(H_2, K)$. For the first $n_1 = k / R(H_1, K)$ channel uses, the transmitter uses the covariance matrix $K$. After these uses, the first user already has enough mutual information to decode the message, and may tune out. Once only the second user is online, the transmitter switches to the optimal $K_2$. With this,

$$n_2 - n_1 = \frac{k - n_1 R(H_2, K)}{C_2}$$

additional uses are needed until the second user has enough information as well. We see, then, that the only compromise is in the choice of covariance matrix for the first period; given this choice, each receiver is able to use all the mutual information provided by the channel, as if it were in a point-to-point scenario. Unfortunately, this information-theoretic result does not tell us how to achieve these rates using *practical* codes; in the sequel, we construct schemes with reduced complexity which use scalar AWGN codes along with linear processing and successive interference cancellation (SIC).

*Remark 1:* We note that the problem simplifies significantly if we constrain the transmitter to always use a white input. In

[1]Note that this scheme assumes a degraded relay channel, namely that the relay is able to recover the message prior to the receiver; otherwise, the receiver is able to decode the message before the relay, and this scheme reduces to that of a "silent" relay throughout the whole transmission process.

this case, the achievable rates, according to Proposition 1, are those satisfying:

$$R_i \leq R\left(H_i, \frac{P}{N_t} I_{N_t}\right) = \log \left| I_{N_t} + \frac{P}{N_t} H_i H_i^\dagger \right| .$$

Thus, we conclude that these white-input rates are always achievable in the rateless setting. In the high-SNR limit (i.e., for fixed non-singular channel matrices where $P \to \infty$), these rates are optimal. For other SNRs, however, such inputs are not optimal and other covariance matrices in Proposition 1 need to be considered.

## III. Codes for Common-Message Multicasting

In this section we describe the main tool which is used in this work. We follow [17] where a practical coding scheme is introduced for a related problem, where data needs to be multicasted to two users using the same effective rate.

We start with $N_t$ codebooks, each one of them good for a SISO Gaussian channel of rate $R_j$ to be determined. At each time instance we form a vector $\tilde{\boldsymbol{x}}$ using one sample from each codeword. The transmitted vector is given by

$$\boldsymbol{x} = K^{1/2} V \tilde{\boldsymbol{x}} , \tag{4}$$

where $E\left\{\tilde{\boldsymbol{x}}^\dagger \tilde{\boldsymbol{x}}\right\} = 1$ and $V$ is unitary, and hence the power constraint is satisfied. Receiver $i$ ($i = 1, 2$) computes[2]

$$\tilde{\boldsymbol{y}}_i = U_i^\dagger \boldsymbol{y}_i , \tag{5}$$

and then decodes the $N_t$ codes using SIC, starting from $\tilde{x}_{N_t}$, where $\tilde{x}_i$ denotes the $i$-th entry of $\tilde{\boldsymbol{x}}$. With respect to the virtual MIMO channel from $\tilde{\boldsymbol{x}}$ to $\tilde{\boldsymbol{y}}_i$, define the successive-decoding signal-to-interference plus noise ratio (SINR) for the $j$-th symbol as:

$$S_{i,j} = \mathrm{Var}\left(\tilde{\boldsymbol{x}}_{\mathrm{j}} \middle| \tilde{\boldsymbol{y}}_{\mathrm{i}}, \tilde{\boldsymbol{x}}_{\mathrm{j+1}}^{\mathrm{N_t}}\right) . \tag{6}$$

The following theorem, due to [17], shows that this strategy is optimal.

*Theorem 1:* For any two channel matrices $H_1$ and $H_2$, and input covariance matrix $K$ such that $R(H_1, K) = R(H_2, K) = R$ there exist $U_1, U_2$ and a unitary $V$ such that the SINRs $S_{i,j}$ (6) satisfy:

$$S_{1,j} = S_{2,j} \qquad \forall j = 1, \ldots, N_t$$
$$\sum_{j=1}^{N_t} \log(1 + S_{1,j}) = R .$$

By the first equation, codebooks of rate $R_j = \log(1 + S_{1,j})$ can be decoded by both receivers; by the second equation, the sum of these rates equals the optimum.

*Remark 2:* Using an input covariance matrix $K$ over the channels described by the channel matrices $H_1$ and $H_2$, is mathematically equivalent to working with a unit covariance matrix $I_{N_t}$ over equivalent channel matrices $F_1 \triangleq H_1 K^{1/2}$ and $F_2 \triangleq H_2 K^{1/2}$, respectively.

[2] $U_i$ $(i = 1, 2)$ are not necessarily unitary.

## IV. Two-user Rateless MIMO Scheme

Let us first restrict attention to integer ratios between the effective rates, i.e., assume that for the covariance matrix $K$ used, there exists an integer $m \geq 2$, such that

$$n_2 = m \cdot n_1 . \tag{7}$$

We can look at the $n_2$ channel uses as $n_1$ uses of the equivalent channels, represented by the block-diagonal matrices:

$$\mathcal{H}_1 = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} H_2 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_2 \end{bmatrix},$$

where $\mathcal{H}_i$ is of dimensions $mN_r^{(i)} \times mN_t$ $(i = 1, 2)$.

Note that if for the equivalent ("augmented") channel input, denoted by $\boldsymbol{\chi}$, we take a covariance matrix

$$\mathcal{K} = \begin{bmatrix} K & 0 & \cdots & 0 \\ 0 & K_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_2 \end{bmatrix},$$

then using (7) and (3), the effective rates of these equivalent channels are *equal* (see (3), i.e.,

$$R \triangleq R(\mathcal{H}_1, \mathcal{K}) = R(\mathcal{H}_2, \mathcal{K}) . \tag{8}$$

Consequently, we can apply the multicast scheme of Theorem 1 to the augmented matrices $\mathcal{H}_1$ and $\mathcal{H}_2$ and input covariance matrix $\mathcal{K}$, or alternatively to the equivalent channel matrices $\mathcal{F}_1 \triangleq \mathcal{H}_1 \mathcal{K}^{1/2}$ and $\mathcal{F}_2 \triangleq \mathcal{H}_2 \mathcal{K}^{1/2}$ with unit covariance matrix (see Remark 2), such that, the optimum rates, as given by Proposition 1, are achieved.

*Scheme 1:*

1) Construct $mN_t$ optimal codes of length $n_1$ for scalar AWGN channels with SNRs equal to the SINRs in (6) and at each augmented channel use form a vector $\tilde{\boldsymbol{\chi}}$ using one sample from each codeword.
2) Combine all these codewords by multiplying $\tilde{\boldsymbol{\chi}}$ by the unitary matrix $V$ of (4) of dimensions $mN_t \times mN_t$:
$$\boldsymbol{\chi} = \mathcal{K}^{1/2} V \tilde{\boldsymbol{\chi}} . \tag{9}$$
3) At the first $n_1$ time instances, transmit a sub-vector composed of the first $N_t$ entries of the corresponding vector $\boldsymbol{\chi}$.
4) The first user, after receiving the first $n_1$ output vectors, multiplies them by $\tilde{U}_1$ of dimensions $mN_t \times N_r^{(1)}$, which is equal to the first $N_r^{(1)}$ columns of $U_1$ of (5). [3] Finally, it decodes the codewords using SIC, resulting in the performance of (6).
5) The second user, continues to receive the rest of the $(n_2 - n_1)$ transmitted vectors. It then interleaves the received output vectors in the following way: It divides

[3] The other columns of $U_1$ must be all-zero since the corresponding channel outputs are noise only, thus the size can be reduced.

all of its $n_2$ received vectors into blocks of length $n_1$, and takes the first vector of each block to construct the first equivalent ("augmented") output vector, then takes the second output vector of each block, etc. It then multiplies each such equivalent vector by $U_2$ of dimensions $mN_t \times mN_r^{(2)}$, and finally decodes the codewords using SIC, resulting in the performance of (6).

*Remark 3:* This scheme can be generalized from integer ratios to any rational ratio between the effective rates, by taking an appropriate numbers of (non-zero) blocks on each of the augmented matrices.

*Remark 4:* A similar scheme can be constructed for varying (*known*) blocks on the diagonal of $\mathcal{H}_2$. This becomes useful for certain cases, e.g., the "half-duplex" relay treated in Section VI. In the case of different blocks on the diagonal, however, the covariance blocks in $\mathcal{K}$ will vary as well. Yet, even in this more general case, only fixed-rate scalar AWGN codebooks need to be employed.

In the special case of the rateless problem where the channel matrix is equal for all users, up to a multiplicative factor $\alpha$:

$$\boldsymbol{y} = \alpha H \boldsymbol{x} + \boldsymbol{z}\,, \tag{10}$$

whereas $\alpha$ is known to the receiver but not to the transmitter.[4], Hybrid ARQ may be used as a means to obtain efficient (rather than choosing the rate conservatively) transmission. Since the channel matrix is known up to a constant factor, one might think that applying the singular-value decomposition (SVD) to that matrix provides a good solution. However, using the SVD results in virtual AWGN channels the gains of which are different in general (being equal to the singular values of $H$). In our case, $H$ is multiplied by $\alpha$, thus multiplying all the virtual scalar channel gains by the *same factor*. Unfortunately, in order to design a perfect rateless scheme for a specific set of ratios between the block lengths (e.g., $m$ in (7), for two blocks), a specific set of ratios between the possible channel gains exists, which *depends on the first (possible) channel gain*; see [4]. A better approach would be to transform the channel into one where all sub-channel gains are equal, thus solving the difficulty above. Such a transformation is offered by the geometric mean decomposition (GMD) [20], which decomposes the matrix using unitary transformations, to achieve a triangular matrix with a *constant* diagonal. This solution still suffers from two drawbacks. First, the GMD is only optimal in the high-SNR limit. Indeed, a variant named uniform channel decomposition (UCD) [18] exists, but it requires knowledge of the SNR and thus cannot be implemented when the gain $\alpha$ is unknown. Second, it still requires to implement a SISO rateless scheme over each of the sub-channels; the design of good rateless codes for the AWGN channel is still an open problem in general, see Section V in the sequel. In the case where the gain $\alpha$ is known to be one of two values (either $\alpha_1$ or $\alpha_2$), we can overcome the difficulties described above by directly applying Scheme 1. Thus we have an *optimal* and *practical* solution for the two-user case.

[4]This is equivalent to not knowing the white noise level $1/|\alpha|^2$.

## V. Application to SISO Rateless Coding

In this section we show that the approach in this work may be helpful even outside the MIMO setting. Consider the case of a SISO channel, i.e., the channel matrices in (1) reduce to a scalar. This always falls under the "unknown SNR" category (discussed in IV), and without loss of generality we take $H = 1$ in (10). This problem is closely related to that of rateless coding for scalar Gaussian channels considered in [4].

In [4], the channel is given by:

$$y = \alpha x + z$$

where $\alpha$ takes one of the values $\alpha_1, \alpha_2, \ldots$, where

$$i \cdot \log(1 + |\alpha_i|^2) = R\,.$$

A practical scheme based on linear pre- and post-processing and SIC was proposed for values $\alpha_1, \ldots, \alpha_M$, and it was shown that a perfect solution (i.e. capacity-achieving) exists for $M = 2$ for any $R$, or for $M = 3$ up to some critical value of $R$.

The case $M = 2$ falls under the category of channels addressed in Section IV, with the augmented matrices being

$$\mathcal{H}_1 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{bmatrix}.$$

Interestingly, the explicit computation of the matrices $U_1, U_2, V$ of that scheme exactly coincide with the matrices derived in [4]. However, our approach provides a straightforward extension to other cases, with either two channel uses or more, as long as the channel materialization is known to be one of two options. Specifically, let $\alpha_{1,1}, \ldots, \alpha_{1,M}$ and $\alpha_{2,1}, \ldots, \alpha_{2,M}$ be the sequences of SNRs according to both options, such that:

$$\sum_{i=1}^{M} \log(1 + \alpha_{1,i}) = \sum_{i=1}^{M} \log(1 + \alpha_{1,i}) = R.$$

Then our scheme achieves exactly the optimum rate $R$. This holds also in the specific case where the trailing SNRs of one of the sequences are zero, and hence applies to the two-option rateless coding problem.

## VI. Application to Half-Duplex Relay

In this section we show how the proposed MIMO rateless scheme of IV yields a practical scheme for the Gaussian MIMO relay problem, in the half-duplex mode. This channel, depicted in Figure 1, is described as follows. The transmitter sends a signal to both the receiver and the relay. In addition, the relay transmits a signal to the receiver. The transmitted signals pass through Gaussian MIMO channels. This channel, depicted in Figure 1, is described by:

$$\begin{aligned} \boldsymbol{y}_{\text{rel}} &= H_{t,\text{rel}} \boldsymbol{x}_t + \boldsymbol{z}_{\text{rel}}\,, \\ \boldsymbol{y}_r &= H_{t,r} \boldsymbol{x}_t + H_{\text{rel},r} \boldsymbol{x}_{\text{rel}} + \boldsymbol{z}_r\,, \end{aligned}$$

where we denote channel output vectors (of lengths $M$) by $\boldsymbol{y}$, and channel input vectors of lengths $N$ – by $\boldsymbol{x}$, using subscripts $r$, $t$ and rel to indicate 'receiver', 'transmitter' and 'relay', respectively. The channel inputs are subject to the same power constraint $P$;[5] we denote the channel matrices of dimensions $M \times N$ by $H$ with two subscripts, where the first (left) subscript indicates the channel input ("originating node"), and the second (right) subscript indicates the channel output ("destination node").

As explained in the introduction, in the half-duplex mode, the relay is restricted to transmitting or receiving at each time instance, but cannot do both. Thus, the transmission is broken into two phases where in the first the relay "listens", until it is able to recover the transmitted message, whereas during the second phase it collaborates with the transmitter, to achieve better reception quality.

Denote by $K_1$ the covariance matrix used by the transmitter during the first transmission phase, lasting $n_1$ time instances, and by $K_2$ – that of the second phase, which lasts $n_2 - n_1$ (the total transmission length is $n_2$).

Since, during the second transmission phase, there are two distinct encoders generating signals which pass through different channels (before being summed), one may attain a greater "coherence gain" by applying, in addition to the "coloring" by non-white covariance matrices, a unitary transformation $\tilde{V}_{\text{rel}}$ at the relay. The improvement due to the introduction of an additional unitary transformation at the relay is apparent in the scalar (SISO) case: In this case, the gains have different phases, and by applying a unitary transformation at the relay (corresponding to multiplying by a phase factor in the scalar case), the phases of both the encoder and the relay can be aligned to attain coherent combining.

Assume for simplicity that for the chosen covariance matrices $K_1$, $K_2$ and $K_{\text{rel}}$ and unitary matrix $\tilde{V}_{\text{rel}}$, $n1 = m \cdot n_2$. This can be generalized in a straightforward manner to any rational ratio, as explained in Remark 3.

Thus, by applying Scheme 1 with the equivalent matrices:

$$\mathcal{F}_1 = \begin{bmatrix} H_{t,\text{rel}} K_1^{1/2} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\mathcal{F}_2 = \begin{bmatrix} H_{t,r} K_1^{1/2} & 0 & \cdots & 0 \\ 0 & \begin{matrix} H_{\text{rel},r} K_{\text{rel}}^{1/2} \tilde{V}_{\text{rel}} \\ + H_{t,r} K_2^{1/2} \end{matrix} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \begin{matrix} H_{\text{rel},r} K_{\text{rel}}^{1/2} \tilde{V}_{\text{rel}} \\ + H_{t,r} K_2^{1/2} \end{matrix} \end{bmatrix},$$

we achieve a rate of $R = \min_{i=1,2} \log \left| I + \mathcal{F}_i \mathcal{F}_i^\dagger \right|$ (see also Remark 4).

Thus, by optimizing over all admissible covariance matrices $K_1$, $K_2$ and $K_{\text{rel}}$ (satisfying the power constraints), and unitary matrices $\tilde{V}_{\text{rel}}$ at the relay, along with matching the appropriate ratios between $n_2$ and $n_1$, a rate of

$$R = \max_{\mathcal{K}} \min_{i=1,2} \log \left| I + \mathcal{F}_i \mathcal{F}_i^\dagger \right| \qquad (11)$$

can be approached, where $\mathcal{F}_1$ and $\mathcal{F}_2$ have $n_1$ and $n_2$ number of blocks on their diagonals, respectively. Thus, we constructed a scheme that uses only linear transformations, SIC and scalar AWGN codes, which approaches the rate (11).

Finally, note that this scheme proves useful even in the SISO ("scalar") case, as it allows the construction of good *practical* transmission schemes, avoiding the need of special codes and codes for channels with varying SNR.

[5] We assume, w.l.o.g., that both $\boldsymbol{x}_t$ and $\boldsymbol{x}_{\text{rel}}$ are subject to the same power constraint, as any difference may be absorbed in the channel-matrices.

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