

Quasi Light Fields: A Model of Coherent Image Formation

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Abstract: We develop a model of coherent image formation that strikes a balance between the simplicity of the light field and the comprehensive predictive power of Maxwell's equations, by extending the light field to coherent radiation.

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1. Introduction

The light field represents radiance as a function of position and direction, thereby decomposing optical power flow along rays. The light field is an important tool in incoherent imaging applications, where it is used to dynamically generate different viewpoints for computer graphics rendering, computational photography, and 3D displays [1–3]. Coherent imaging researchers have recognized the value of decomposing power by position and direction without the aid of a light field, because the complex-valued scalar field encodes direction in its phase, representing multiple viewpoints in holography, ultrasound, and optical system modeling. Judging by these applications, coherent imaging uses the scalar field to achieve results similar to those that incoherent imaging obtains with the light field [4–6].

Our goal is to provide a model of coherent image formation that combines the utility of the light field with the comprehensive predictive power of the scalar field. The similarities between coherent and incoherent imaging motivate exploring how the scalar field and light field are related, which we address by synthesizing research in optics, quantum physics, and signal processing. Our contribution is to describe and characterize all the ways to extend the light field to coherent radiation to form quasi light fields, and to interpret coherent image formation using the quasi light fields.

The traditional light field for incoherent radiation acts as an intermediary between the camera and the picture, decoupling information capture and image production: the camera measures the light field, from which many different traditional pictures can be computed. We define an image pixel value as the power P radiated by a patch σ on the surface of the scene towards a virtual aperture (Figure 1). According to radiometry, P is obtained by integrating over a bundle of light field rays [7]:

$$P = \int_{\sigma} \int_{\Omega_r} L(\mathbf{r}, \mathbf{s}) \cos \psi d^2s dr, \quad (1)$$

where $L(\mathbf{r}, \mathbf{s})$ is the radiance at position \mathbf{r} and in unit direction \mathbf{s} , ψ is the angle that \mathbf{s} makes with the surface normal at \mathbf{r} , and Ω_r is the solid angle subtended by the virtual aperture at \mathbf{r} . The light field is constant along rays in a lossless medium, so that we may infer the light field on the scene surface by remotely intercepting the desired rays. Thereby, the images produced by many different conventional cameras can be computed from the light field L using (1) [8].

In order to generate coherent images using the same framework described above, we first determine how to measure power flow by position and direction to formulate the quasi light fields (Section 2), and then capture quasi light fields remotely so as to infer behavior at the scene surface (Section 3). Finally, we use (1) to produce physically correct power values, so that we form images by integrating over quasi light fields (Section 4).

2. Formulating Quasi Light Fields

A quasi light field must produce physically accurate power transport predictions; thus the power computed from the scalar field using wave optics must match the power computed from a quasi light field using the laws of radiometry.

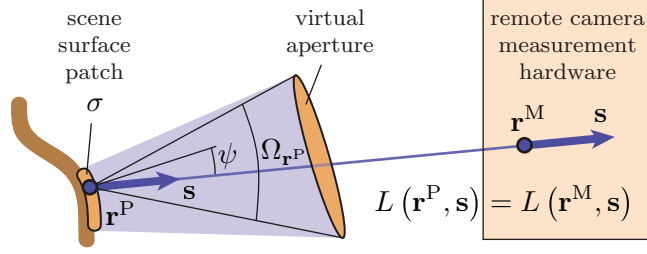


Fig. 1. We compute the value of each pixel in an image produced by an arbitrary virtual camera, defined as the power radiating from a scene surface patch towards a virtual aperture, by integrating an appropriate bundle of light field rays that have been previously captured with remote hardware.

Reasoning in this way, Walther identified the first quasi light field [9]

$$L^W(\mathbf{r}, \mathbf{s}) = \left(\frac{k}{2\pi}\right)^2 s_z \int U\left(\mathbf{r} + \frac{1}{2}\mathbf{r}'\right) U^*\left(\mathbf{r} - \frac{1}{2}\mathbf{r}'\right) \exp(-iks_{\perp} \cdot \mathbf{r}'_{\perp}) d^2r', \quad (2)$$

where U is the scalar field, k is the wave number, and \mathbf{r}_{\perp} and \mathbf{s}_{\perp} denote the projections of \mathbf{r} and \mathbf{s} onto the xy -plane.

We synthesize research in quantum physics and signal processing to systematically generate and characterize all coherent light fields. After recognizing (2) as the Wigner quasi-probability distribution, Agarwal et al. repurposed the mathematics of quantum physics to generate a class of coherent light fields [10]. We express these *quasi light fields* as filtered Wigner distributions, which are known in the signal processing community as shift-invariant bilinear forms of U and U^* , comprising the *quadratic class* of time-frequency distributions [11]. The quadratic class contains all reasonable ways of distributing energy over position and direction, thereby containing all coherent light fields. Members of the quadratic class have been extensively characterized, enabling us to select an appropriate quasi light field for a specific application [11, 12].

3. Capturing Quasi Light Fields

While the traditional light field is non-negative and typically captured directly by making intensity measurements at a discrete set of positions and directions, many quasi light fields are negative or complex-valued, and therefore cannot be similarly captured. Instead, we capture quasi light fields by sampling the scalar field with a discrete set of sensors, and subsequently processing the scalar field measurements to compute the desired quasi light field. For simplicity, we consider a two-dimensional scene and a linear array of sensors regularly spaced along the y -axis.

Choosing a quasi light field is an exercise in managing tradeoffs, as no quasi light field has all the properties of the traditional light field, which is non-negative in general and zero where the scalar field is zero [13]. We compare three discrete quasi light fields; ignoring constants and s_z , the spectrogram ℓ^S is non-negative, the Rihaczek ℓ^R is zero where the scalar field is zero, and the Wigner ℓ^W possesses neither property:

$$\begin{aligned} \ell^S(y, s_y) &= \left| \sum_n T(nd) U(y + nd) \exp(-iknds_y) \right|^2, \\ \ell^R(y, s_y) &= U^*(y) \sum_n U(nd) \exp(-iknds_y), \\ \ell^W(y, s_y) &= \sum_n U(y + nd/2) U^*(y - nd/2) \exp(-iknds_y). \end{aligned} \quad (3)$$

Different quasi light fields localize energy in position and direction in different ways, so that the choice of quasi light field impacts the potential resolution achieved in an imaging application, which we illustrate by simulating a plane wave propagating past a screen edge and comparing ℓ^S , $|\ell^R|$, and $|\ell^W|$ (Figure 2). Each quasi light field encodes both the position of the screen edge and the downward direction of the plane wave, but the spectrogram doesn't resolve the plane wave as well as the others. Since non-negative quasi light fields are sums of spectrograms, directly capturing light fields by making non-negative intensity measurements evidently limits energy localization possibilities.

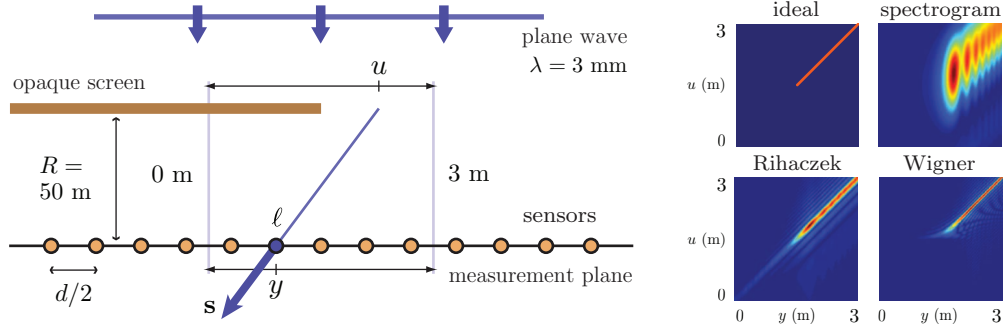


Fig. 2. Compared to the spectrogram, other quasi light fields, such as the Rihaczek and Wigner, better resolve a plane wave propagating past the edge of an opaque screen. The ringing and blurring in the light field plots indicate the diffraction fringes and energy localization limitations, respectively. We use a 10 cm rectangular window for T in (3).

4. Image Formation

We form images from quasi light fields for coherent applications similarly to how we form images from the traditional light field for incoherent applications, by integrating bundles of light field rays to compute pixel values. This raises two questions: how do we relate the choice of quasi light field to image formation, and is the resulting image physically accurate? First, although *power radiating from a scene surface patch towards an aperture* has a unique interpretation for incoherent light of a small wavelength, it is ambiguous for coherent light according to the classical wave theory, requiring a choice of quasi light field to resolve the ambiguity and precisely define an image. Second, all quasi light fields implicitly make the far-zone (Fraunhofer) diffraction approximation. To correct for inaccuracies in the near zone, we may augment quasi light fields with a parameter to track the distance between the source and point of observation along each ray.

Aside from the far-zone approximation, image formation from quasi light fields is accurate and generalizes the classic beamforming algorithm used in sensor array processing [5]. For example, the pixel power according to the Rihaczek light field in the far zone P^R factors into the Hermitian product of two different beamformers g_1 and g_2 ,

$$P^R \propto \left[\sum_{|md| < A/2} U(md) \exp(-ikmds_y) \right]^* \left[\sum_n U(nd) \exp(-iknds_y) \right] \triangleq g_1^* g_2, \quad (4)$$

where the virtual aperture has width A and lies along direction \mathbf{s} from the scene surface patch. Beamformer g_1^* aggregates power contributions across the aperture using measurements of the conjugate field U^* on the aperture, while beamformer g_2 isolates power from the scene surface patch using all available measurements of the field U . The tasks of aggregating and isolating power contributions are cleanly divided between the two beamformers, and each beamformer uses the measurements from those sensors appropriate to its task. Image formation with alternative quasi light fields uses the conjugate field and field measurements to aggregate and isolate power in different ways.

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