On the Capacity Region of Asynchronous Channels

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***Abstract*—We consider asynchronous communication over discrete memoryless channels. The transmitter starts sending one block codeword of length N at an instant that is uniformly distributed within a certain time period A, which represents the level of asynchronism between the transmitter and the receiver. The receiver, by means of a sequential decoder, must isolate the message without knowing when the codeword transmission starts but being cognizant of the asynchronism level. Motivated by certain monitoring type of applications, we are interested in communication strategies that 1) operate with short codeword length with respect to the asynchronism level and 2) that guarantee quick decoding.**

In a recent work the authors showed that the communication rate — defined with respect to the decoder's reaction delay to the sent message — can be strictly positive unless A grows faster than $e^{N\alpha}$ and α exceeding the *synchronization threshold*.

The present work focuses on the regime where α is smaller than the synchronization threshold. The main contribution consists of simple expressions that give upper and lower bounds on the highest achievable rate for any α below the synchronization threshold. For random code constructions these bounds are tight.

***Index Terms*—Asynchronous communication, detection and isolation problem, discrete-time communication, error exponent, monitoring, point-to-point communication, quickest detection, sequential analysis, sparse communication, stopping times**

I. Introduction

Information theoretic communication models commonly assume that the transmitter and the receiver are perfectly synchronized. Basic quantities, such as the channel capacity, are defined under this assumption [3]. Coding strategies need only to overcome the 'vertical' uncertainty introduced by the channel noise, but not the 'horizontal' uncertainty introduced when the receiver has only a partial knowledge of *when* information is sent. However, this assumption appears unreasonable for many situations, e.g., when there is time uncertainty due to burstiness in the information source. In such cases, an inherently asynchronous communication model is more appropriate.

The present work is motivated by questions such as 'What is the rate loss incurred by lack of synchronization?', 'What is the tradeoff between asynchronism level and the maximum rate for which reliable communication can still be achieved?', 'How to code over asynchronous channels?.' To provide some answers to these questions, we pursue the investigation of the communication model introduced in [4]. This model is motivated by certain monitoring type of applications where a sensor sporadically emits alarm messages to a base station, and otherwise remains idle. The time an alarm is sent relates to external events (such as fire) which results in asynchronous communication between the sensor and the base station. In such a setting, two communication parameters are important: the message size and the 'reaction delay' of the base station to the sent message. The message size, assuming the same cost per transmitted symbol, should be minimized in order to meet low sensor energy consumption criteria. Similarly, the reaction delay to the sent message should also be minimized in order to allow quick responses to the alarms.

The model assumes that the transmitter starts sending information at a time that is unknown to the receiver and uniformly distributed within an interval of known size, which defines the asynchronism level between the transmitter and the receiver. The communication rate is defined with respect to the average reaction delay, and we aim to maximize it given the asynchronism level.

In the next section we recall the formulation of the problem introduced in [4].

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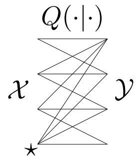

Fig. 1. Communication is carried over a discrete memoryless channel. When 'no information' is sent the input of the channel is the '$\star$' symbol.

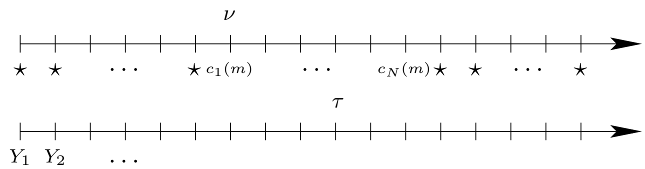

Fig. 2. Time representation of what is sent (upper arrow) and what is received (lower arrow). The '$\star$' represents the 'noise' symbol. At time ν message m starts being sent and decoding occurs at time τ.

II. Problem formulation

We first introduce the communication model and its performance criteria, then comment on them.

We consider discrete-time communication over a point-to-point discrete memoryless channel (DMC) characterized by its finite input and output alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively, transition probability matrix $Q(y|x)$, for all $y \in \mathcal{Y}$ and $x \in \mathcal{X}$, and 'noise' symbol $\star \in \mathcal{X}$ (see Fig. 1).[1] The codebook $\mathcal{C}$ consists of $M \geq 2$ equally likely codewords of length N composed of symbols from $\mathcal{X}$ — possibly also the $\star$ symbol. The transmission of a particular codeword starts at a random time ν, independent of the codeword to be sent, uniformly distributed in $[1, 2, \ldots, A]$, where the integer $A \geq 1$ characterizes the asynchronism level between the transmitter and the receiver. We assume that the receiver knows A but not ν. If $A = 1$ the channel is said to be synchronized.

Before and after the transmission of the information, i.e., before time ν and after time $\nu + N - 1$, the receiver observes noise. Specifically, conditioned on the value of ν and on the message to be conveyed m, the receiver observes independent symbols $Y_1, Y_2, \ldots$ distributed as follows. If $i \leq \nu - 1$ or $\nu + N \leq i \leq A + N - 1$, the distribution is $Q_\star \triangleq Q(\cdot|\star)$. At any time $i \in [\nu, \nu + 1, \ldots, \nu + N - 1]$ the distribution is $Q(\cdot|c_{i-\nu+1}(m))$, where $c_n(m)$ denotes the nth symbol of the codeword $c^N(m)$ assigned to message m.

The decoder consists of a sequential test (τ, ϕ), where τ is a stopping time — bounded by $A + N - 1$ — with respect to the output sequence $Y_1, Y_2, \ldots$[2] indicating when decoding happens, and where ϕ denotes a decision rule that declares the decoded message (see Fig. 2).[3]

We are interested in *reliable and quick decoding*. To that aim we first define the maximum (over the messages) decoding error probability as

$$\mathbb{P}(\mathcal{E}) = \max_m \frac{1}{A} \sum_{l=1}^{A} \mathbb{P}_{m,l}(\mathcal{E}),$$

where $\mathcal{E}$ indicates the event that the decoded message does not correspond to the sent codeword, and where the subscripts $_{m,l}$ indicate the conditioning on the event that message m starts being sent at time $\nu = l$. Second, we define the communication rate with respect to the maximum (over messages) communication delay it takes the receiver to react to the sent codeword, i.e.

$$R = \frac{\ln M}{\mathbb{E}(\tau - \nu)^+} \quad \text{nats per channel use,} \tag{1}$$

where

$$\mathbb{E}(\tau - \nu)^+ = \max_m \frac{1}{A} \sum_{l=1}^{A} \mathbb{E}_{m,l}(\tau - l)^+ \ ,$$

x^+ denotes $\max\{0, x\}$, and where $\mathbb{E}_{m,l}$ denotes the expectation with respect to $\mathbb{P}_{m,l}$.

As shown in [4, Theorem 1], the exponential growth of the asynchronism level with respect to the codeword length represents a natural scaling: vanishing error probability can be achieved only if A grows no faster than exponentially with N with an exponent smaller than the synchronization threshold. This motivates the following definition.

Definition 1 **((R, α) coding scheme).** *A pair (R, α) is achievable if there exists a sequence $\{(\mathcal{C}_N, (\tau_N, \phi_N)\}_{N \geq 1}$ of codebook/decoder pairs, labeled by the codebook length N, that asymptotically achieves a rate R at an asynchronism exponent α. This means that, for any $\varepsilon > 0$ and N large enough, the pair $(\mathcal{C}_N, (\tau_N, \phi_N))$*

- *operates under asynchronism level $A = e^{(\alpha - \varepsilon)N}$;*
- *yields a rate at least equal to $R - \varepsilon$;*
- *achieves a maximum error probability at most equal to ε.*

An (R, α) coding scheme is a sequence $\{(\mathcal{C}_N, (\tau_N, \phi_N))\}_{N \geq 1}$ that achieves a rate R at an asynchronism exponent α as $N \to \infty$.

[1] Throughout the paper we always assume that for all $y \in \mathcal{Y}$ there is some $x \in \mathcal{X}$ for which $Q(y|x) > 0$.

[2] Recall that a (deterministic or randomized) stopping time τ with respect to a sequence of random variables $\{Y_i\}_{i=1}^{\infty}$ is a positive, integer-valued random variable such that the event $\{\tau = n\}$ depends only on $Y_1, Y_2, \ldots, Y_n$ and not on $Y_{n+1}, Y_{n+2}, \ldots$, for all $n \geq 1$.

[3] Formally, ϕ is an $\mathcal{F}_\tau$-measurable map ($\mathcal{F}_1, \mathcal{F}_2, \ldots$ is the natural filtration induced by the process $Y_1, Y_2, \ldots$) with $\phi(y^\tau) \in \{1, 2, \ldots, M\}$ for any y^τ.

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Definition 2 (capacity region of an asynchronous DMC**).** *The capacity region of an asynchronous DMC with synchronized capacity $C(Q)$ is the function*

$$[0, C(Q)] \to \mathbb{R}_+$$
$$R \mapsto \alpha(R, Q)$$

where $\alpha(R, Q)$ is the supremum of the set of asynchronism exponents that are achievable at rate R.

We now motivate our model and its performance criteria. For a detailed discussion we refer the reader to [4].

First observe that there is no feedback from the receiver to the transmitter. With noiseless feedback it is possible to inform the transmitter of the receiver's decoding time, say in the form of ACK/NACK, therefore allowing the sending of multiple messages instead of just one as in our model. Here the noiseless assumption is crucial. If the feedback is noisy, the receiver's decision may be wrongly recognized by the transmitter, which possibly may result in a loss of message synchronization between transmitter and receiver (say the receiver hasn't yet decoded the first message while the transmitter has already started to emit the second one). Therefore, in order to avoid a potential second source of asynchronism, we omit feedback in our study and limit transmission to only one message.

The reaction delay $\mathbb{E}(\tau - \nu)^+$ indicates the average time the transmitter needs to wait until the receiver makes a decision. It is therefore reasonable to define the communication rate with respect to it instead of with respect to the codeword length N as usually. Indeed, the blocklength does not have a direct operational meaning: in light of the use of sequential decoding, the codeword length does not provide a measure of the delay needed for the information to be reliably decoded. Instead of $\mathbb{E}(\tau - \nu)^+$ one might also consider defining the rate with respect to $\mathbb{E}(\tau)$ or, equivalently, $\mathbb{E}\nu + \mathbb{E}(\tau - \nu)$. The fact that this delay takes into account the initial offset $\mathbb{E}\nu$ can be regarded as a weakness since it can be influenced neither by the transmitter nor by the receiver. Also, were we to choose such a delay measure, it can be shown that, in the regime of positive asynchronism exponents in which we are interested, the achievable rate would always be (asymptotically) vanishing for any reliable coding strategy.

In the definition of achievable pair (R, α) (Definition 1), we choose to grow A with N. Indeed, when A is fixed the problem of finding the capacity region of asynchronous channels becomes trivial since it reduces to the computation of the capacity of the synchronized channel [4].

III. Results

We start with a few notation conventions. We denote by $\mathcal{P}_X$ and $\mathcal{P}_Y$ the set of distributions over the finite alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively, and by $\mathcal{P}_{Y|X}$ the set of conditional distributions over $\mathcal{X} \times \mathcal{Y}$. We use $(PQ)_Y(y)$ to denote $\sum_x P(x)Q(y|x)$ for some $P \in \mathcal{P}_X$ and $Q \in \mathcal{P}_{Y|X}$. Also, we denote by $I(PQ)$ the mutual information induced by the joint distribution $P(\cdot)Q(\cdot|\cdot)$. The Kullback-Leibler distance between two distributions P_1 and P_2 is denoted by $D(P_1||P_2)$. The following Theorem provides an inner bound on the capacity region of asynchronous DMCs.

Theorem 1. *Let $P \in \mathcal{P}_X$ be such that, for any distribution $V \in \mathcal{P}_Y$, at least one of the following inequalities*

$$D(V||(PQ)_Y) > \alpha$$
$$D(V||Q_\star) > \alpha$$

holds, i.e.

$$\min_{V \in \mathcal{P}_Y} \max\{D(V||(PQ)_Y), D(V||Q_\star)\} > \alpha\ .$$

Then, the pair $(R = I(PQ), \alpha)$ is achievable.

Hence, maximizing over the input distributions, a lower bound to $\alpha(R, Q)$ in Definition 2 is

$$\max_{P \in \mathcal{P}_X : I(PQ) \geq R} \min_{V \in \mathcal{P}_Y} \max\{D(V||(PQ)_Y), D(V||Q_\star)\}\ . \tag{2}$$

Sketch of the proof of Theorem 1: The proof is based on a random coding argument associated with the following strategy. Each codeword has a common initial prefix of size (actually $(\ln N)^\rho$ with any $\rho > 1$ works as well) followed by $N - (\ln N)^2$ information symbols. The prefix is a maximum length shift register sequence [2]. The part that carries information is randomly generated so that each symbol of each codeword is i.i.d. according to P.

The sequential decoder operates according to a two-step procedure. The first step consists in making an approximate estimation of the location of the sent codeword by using only the $N - (\ln N)^2$ information symbols of each codeword. It should be emphasized that the prefix is *not* involved in this initial detection phase. Specifically, at time n the decoder tests whether the last $N - (\ln N)^2$ symbols are generated by noise or by some codeword on the basis of their empirical distribution $\hat{P}$. If $D(\hat{P}||Q_\star) \leq \alpha$ the decoder decides to continue to time $n+1$. If $D(\hat{P}||Q_\star) > \alpha$, i.e., if $\hat{P}$ is far from the noise distribution, the decoder marks the current time as the beginning of the 'decoding window' and proceeds to the second step of the decoding procedure. This step consists

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in exactly locating and identifying the sent codeword. Once the beginning of the decoding window has been marked, the decoder stops and makes a decision the first time both the last $N-(\ln N)^2$ symbols are jointly typical with one of the codewords, and the previous $(\ln N)^2$ symbols are jointly typical with the prefix. If no such time is found within N successive time steps, the decoder declares a random message.

The encoding corresponding to the information symbol portion of the codeword must accomplish two objectives. First, it must help the decoder approximate the codeword location, and, second, it must enable reliable decoding of the message conditioned on the decoder knowing the codeword location. The role of the prefix is to allow a second order fine estimation of the location of the sent codeword, once this position has already been approximated using the second part of the codewords.

The theorem is proven by analyzing the above coding strategy and using the fact that a maximum length shift register sequence has the property that the Hamming distance between any two (circular) shifts of the sequence is linear in the sequence length. This property guarantees that, given that the codeword's position has been correctly approximated during the first phase of the decoding procedure, i.e., that the confidence interval for the position is of order N, the prefix identifies the exact position with high probability. ■

The next result provides an outer bound on the capacity region of asynchronous DMCs. Recall that a codebook has constant composition P if all the codewords have the same empirical type [1, p.117].

Theorem 2. *Suppose $\{(\mathcal{C}_N,(\tau_N,\phi_N))\}_{N\geq 1}$ achieves (R,α) with $R>0$. Then there exists $\mathcal{C}'_N\subset\mathcal{C}_N$ such that $\{(\mathcal{C}'_N,(\tau_N,\phi_N))\}_{N\geq 1}$ has the following properties. Each $\mathcal{C}'_N$ has constant composition P^1_N over the first $d_N \triangleq \min\{\mathbb{E}(\tau_N-\nu)^+,N\}$ symbols and constant composition P^2_N over N symbols. Furthermore, $|\mathcal{C}'_N|$, d_N, P^1_N, and P^2_N satisfy*

i) *$I(P^1_N Q)(1+o(1)) \geq \dfrac{\ln|\mathcal{C}'_N|}{d_N} = R(1+o(1))$ as $N\to\infty$;*

ii) *For any $W\in\mathcal{P}_{Y|X}$ at least one of the following two inequalities holds as $N\to\infty$[4]*

$$D(W||Q_\star|P^2_N) > \alpha(1+o(1))$$
$$D(W||Q|P^2_N) > \alpha(1+o(1))\ .$$

If $\mathbb{E}(\tau_N-\nu)^+ = N$, then $P^1_N = P^2_N$. However, if $\mathbb{E}(\tau_N-\nu)^+\neq N$, the theorem does not say how P^1_N and P^2_N are related. The fact that potentially $\mathbb{E}(\tau_N-\nu)^+\neq N$ for capacity achieving strategies represents a major difficulty in deriving the capacity region for asynchronous channels.

Sketch of the proof of Theorem 2: Let $\{(\mathcal{C}_N,(\tau_N,\phi_N))\}_{N\geq 1}$ be a coding scheme that satisfies the hypothesis of the Theorem. An expurgation argument shows that there exists $\{(\mathcal{C}'_N,(\tau_N,\phi_N))\}_{N\geq 1}$ with the following properties. Each $\mathcal{C}'_N$ is a subset of $\mathcal{C}_N$ and has constant composition P^1_N over $d_N\triangleq\min\{\mathbb{E}(\tau_N-\nu)^+,N\}$ symbols and constant composition P^2_N over N symbols. Furthermore, $|\mathcal{C}'_N|$, d_N, and P^1_N, satisfy the condition i) in the theorem statement

$$I(P^1_N Q)(1+o(1)) \geq \frac{\ln|\mathcal{C}'_N|}{d_N} = R(1+o(1))$$

as $N\to\infty$. To show that P^2_N satisfies the condition ii) we proceed as follows.

Consider the maximum (over messages) average reaction delay $\mathbb{E}(\tau_N-\nu)^+$. Let $c^N(m)$ be the codeword assigned to message m, $W\in\mathcal{P}_{Y|X}$, and let $Y^{(i)}$ denote the output sequence of length N running from time i up to time $i+N-1$. Using Markov's inequality and [1, Lemma 2.6, p. 32] one first shows that

$$\begin{aligned}\mathbb{E}(\tau_N-\nu)^+ &\triangleq \max_m \frac{1}{A}\sum_{i=1}^{A}\mathbb{E}_{m,i}(\tau_N-i)^+\\ &\geq \frac{1}{3}\frac{e^{-ND_1}}{(N+1)^{|\mathcal{X}|\cdot|\mathcal{Y}|}}\\ &\quad\times\sum_{i=1}^{A/3}\mathbb{P}_{m,i}(\tau_N\geq i+A/3|Y^{(i)}\in T_W(c^N(m)))\end{aligned}$$

for any m, where $D_1\triangleq D(W||Q|P^2_N)$ and where $T_W(c^N(m))$ denotes the set of sequences y^N that are in the W-shell of the codeword $c^N(m)$, i.e., whose conditional distribution with respect to $c^N(m)$ is W (see [1, p.31]).

The key step is to observe that the change of measure

$$\begin{aligned}&\mathbb{P}_{m,i}(\tau_N\geq i+A/3|Y^{(i)}\in T_W(c^N(m)))\\ &\quad=\mathbb{P}_\infty(\tau_N\geq i+A/3|Y^{(i)}\in T_W(c^N(m)))\end{aligned}\qquad(3)$$

holds, where $\mathbb{P}_\infty$ denotes the output distribution under pure noise, i.e., when the Y_i's are i.i.d. according to $Q_\star$ for all $i\in[1,2,\ldots,A+N-1]$. To see this note that

$$\begin{aligned}&\mathbb{P}_{m,i}(\tau_N\geq i+A/3|Y^{(i)}=y^{(i)})\\ &\quad=\mathbb{P}_\infty(\tau_N\geq i+A/3|Y^{(i)}=y^{(i)})\end{aligned}$$

[4]Given $W_1,W_2\in\mathcal{P}_{Y|X}$ and $P\in\mathcal{P}_X$, $D(W_1||W_2|P)$ denotes the Kullback-Leibler distance between $P(\cdot)W_1(\cdot|\cdot)$ and $P(\cdot)W_2(\cdot|\cdot)$.

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and thus

$$\begin{aligned}
&\mathbb{P}_{m,i}(\tau_N \geq i + A/3 | Y^{(i)} \in T_W(c^N(m))) \\
&= \sum_{y^{(i)} \in T_W(c^N(m))} \mathbb{P}_{m,i}(\tau_N \geq i + A/3 | Y^{(i)} = y^{(i)}) \\
&\qquad \times \mathbb{P}_{m,i}(Y^{(i)} = y^{(i)} | Y^{(i)} \in T_W(c^N(m))) \\
&= \sum_{y^{(i)} \in T_W(c^N(m))} \mathbb{P}_{\infty}(\tau_N \geq i + A/3 | Y^{(i)} = y^{(i)}) \\
&\qquad \times \mathbb{P}_{\infty}(Y^{(i)} = y^{(i)} | Y^{(i)} \in T_W(c^N(m))) \\
&= \mathbb{P}_{\infty}(\tau_N \geq i + A/3 | Y^{(i)} \in T_W(c^N(m))) \ .
\end{aligned}$$

Using the above observation and applying again [1, Lemma 2.6, p. 32] one deduces that

$$\begin{aligned}
\mathbb{E}(\tau_N - \nu)^+ &\geq \frac{e^{-N(D_1 - D_2)}}{3(N+1)^{|\mathcal{X}|\cdot|\mathcal{Y}|}} \\
&\quad \times \sum_{i=1}^{A/3} \mathbb{P}_{\infty}(\tau_N > 2A/3, Y^{(i)} \in T_W(c^N(m)))) \quad (4)
\end{aligned}$$

where $D_2 \triangleq D(W||Q_\star|P_N^2)$. Finally, letting $A = e^{N(\alpha+\mu)}$, where $\mu > 0$ is arbitrarily small, one gets

$$\begin{aligned}
&\sum_{i=1}^{A/3} \mathbb{P}_{\infty}(\tau_N > 2A/3, Y^{(i)} \in T_W(c^N(m))) \\
&\qquad = \Omega(e^{N(\alpha+\mu) - N(D_2 + \mu/2)})
\end{aligned}$$

if $D_2 < \alpha$ by (reverse) union bound. Hence using (4), if $D_1 < \alpha$ and $D_2 < \alpha$, then $\mathbb{E}(\tau_N - \nu)^+$ grows exponentially with N, implying that the rate is asymptotically equal to zero. We conclude that, if the coding scheme achieves a strictly positive rate, then either $D_1 \geq \alpha$, or $D_2 \geq \alpha$, or both inequalities hold. Condition ii) follows. ∎

The following result is related to the *random codebook* capacity region, i.e., the capacity region computed with respect to codebooks having all the components of all the codewords independently drawn according to some common distribution. Specifically, an (R, α) coding scheme $\{(\mathcal{C}_N, (\tau_N, \phi_N))\}_{N \geq 1}$ with random codebooks has all the codebooks $\{\mathcal{C}_N\}_{N \geq 1}$ randomly generated according to some distribution P, and satisfies the important constraint $\lim_{N \to \infty} \mathbb{E}(\tau_N - \nu)^+/N \geq 1$.

Theorem 3. *The random codebook capacity region over strictly positive rates is characterized by* (2).

Sketch of the proof of Theorem 3: Let $\{(\mathcal{C}_N, (\tau_N, \phi_N))\}_{N \geq 1}$ be an (R, α) $(R > 0)$ coding scheme with each codebook $\mathcal{C}_N$ randomly generated according to P.

By Fano's inequality and the constraint $\lim_{N \to \infty} \mathbb{E}(\tau_N - \nu)^+/N \geq 1$ the input distribution P must satisfy $I(PQ) \geq R$. The second argument consists in showing that one of the following inequalities

$$\begin{aligned}
D(V||(PQ)_Y) &> \alpha \\
D(V||Q_\star) &> \alpha
\end{aligned}$$

must hold for any $V \in \mathcal{P}_Y$. One then concludes that α is upper bounded by

$$\max_{P: I(PQ) \geq R} \min_{V \in \mathcal{P}_Y} \max\{D(V||(PQ)_Y), D(V||Q_\star)\} \ ,$$

and, using Theorem 1, one deduces the theorem.

To prove the desired claim one repeats the argument that yields Theorem 2 claim ii) by replacing the set $T_W(c^N(m))$ by $T^N(V)$, the set of sequences $y^N \in \mathcal{Y}^N$ that have empirical type equal to V. In particular note that the key step (3) holds since, under $\mathbb{P}_{m,i}$, the probability of any sequence $y^{(i)}$ conditioned on $T^N(V)$ is the same. ∎

IV. Remark

It may be interesting to note that the achievability result is obtained via a non-standard training based coding strategy. The small prefix at the beginning of each codeword is not used to pinpoint the codeword's location as in practice. Instead, it is the large portion of information symbols that is mainly used for detecting the codeword. These symbols locate the codeword within an order N confidence interval of the exponentially large (in N) uncertainty window. The role of the prefix is only at a second order since it is used to exactly locate the codeword within an order N uncertainty window.

Because random codes achieve the capacity of synchronized channels, it may be tempting to believe that this is also valid in the situation of asynchronous channels. Surprisingly perhaps, this claim is false. Indeed, preliminary results show that non-randomly constructed codebooks — i.e., not satisfying the conditions given in the paragraph that precedes Theorem 3 — outperform the bound (2).

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