Scalable Blind Calibration of Timing Skew in High-Resolution Time-Interleaved ADCs

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Abstract— **The performance of high-resolution time-interleaved analog-to-digital converters is often significantly degraded by timing mismatch errors. This paper examines low-complexity methods for performing blind calibration of such converters. In particular, we develop a least squares formulation for estimating the unknown time-skew parameters and for performing signal reconstruction from these estimates. The complexity of the proposed algorithm scales linearly with the number of converters, making it an attractive solution for calibration. Tradeoffs between performance and complexity are also developed.**

I. Introduction

The maximum sampling rate of single analog-to-digital converters is commonly lower than the rate desired by the current technologies. At these high speeds, time-interleaved analog-to-digital converters (TIADCs) offer an efficient method of sampling by distributing the load across many converters.

TIADCs operate in a round-robin manner. In a system of M converters, to realize a system sampling period of T_s, each converter operates with sampling period MT_s and a spacing of T_s between consecutive converters. Thus, the sampling rate required by the ADCs in the system is reduced by a factor of M, allowing for a lower amount of overall power consumption and a greater control over sampling accuracy.

Although TIADCs may alleviate some of the problems encountered when using a single high speed ADC, they also introduce a new set of problems. In particular, variations among the individual ADCs in a time-interleaved system lead to inaccurate sampling [1]. The primary source of error in high-resolution TIADCs is timing skew. There are also gain and amplitude offset variations among the converters [2]; however a variety of circuit based matching techniques exist for the minimization of such errors. For this reason, we focus on signal recovery when only timing skews are unknown in the system and assume that the gains and amplitudes are calibrated [3]. The general case is examined briefly in this paper and studied in more detail in e.g. [4].

There are two general approaches to system calibration. The first approach is to incorporate a known signal in the input in order to estimate the unknown parameters [5]. The various implementations of such a method may require extra hardware, decrease the sampling resolution, or cause system delays by pausing the normal input signal. The second approach is to perform blind recovery using only the ADC outputs. Such methods may use oversampling and take advantage of the excess bandwidth in the system to enable calibration. Existing blind methods focus on the cases of large amounts of oversampling, stationarity of the inputs, or require a considerably large amount of complexity [6],[7],[8].

In contrast to the approaches above, we note that in typical high-resolution architectures the time-skews are generally small relative to the system sampling period. In this paper, we optimize for the small-error regime to develop a method for blind calibration of TIADCs systems, which has low complexity even for systems with many converters. We present a method that both estimates the converter skews and creates a reconstruction of the input sampled on a uniform grid, provided that the overall system sampling rate is above the Nyquist rate of the input.

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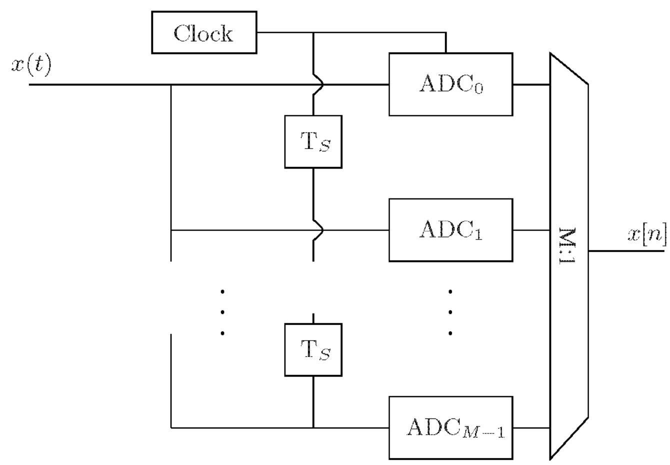

Fig. 1. Ideal time-interleaved ADC system with M converters

II. Problem Formulation

We model the converter input $x(t)$ as a bandlimited signal with cutoff frequency Ω_c, i.e. the continuous time Fourier transform $X(j\Omega) = 0$ for $\Omega_c < |\Omega| \le \pi$. The overall sampling period T_s of the system is chosen to ensure that the sampling rate strictly exceeds the Nyquist frequency, i.e., $T_s < \pi/\Omega_c$, thus creating some amount of excess bandwidth. The signal recovery problem is to estimate $x[n] = x(nT_s)$, which is bandlimited to $\omega_c = \Omega_c T_s < \pi$, as accurately as possible from the ADC outputs.

We model the output of the ith constituent ADC as

$$y_i[n] = x(MT_s n + \tau_i) + w_i[n] \quad (1)$$

where the τ_i model the unknown skews. We also make the assumption that the timing skews are relatively small, e.g., not more than 10% of the overall sampling period. The $w_i[n]$ represent the quantization noise, whose variance depends on the number of bits to which the input is quantized. For ease of analysis, we assume the input is quantized with high-resolution, $w_i[n] \approx 0$; however the effects of quantization noise are considered within the subsequent simulations. Without loss of generality, we can choose an arbitrary time reference, thus we let $\tau_0 = 0$.

III. Small Error Resampling

We use the following notation to represent the signal obtained by multiplexing the ADC outputs

$$y[n] \quad = \quad y_i\left[\frac{n-i}{M}\right] \qquad n \pmod{M} = i. \tag{2}$$

This received signal is also referred to as the uncalibrated signal.

There are multiple methods for signal reconstruction when the timing skews are known. In this paper, we focus on the method developed in [9] to motive our approach. The reconstruction for the M-ADC system is given by

$$x(t) = \gamma(t) \sum_{\alpha=-\infty}^{\infty} \sum_{i=0}^{M-1} y[M\alpha + i] \frac{a_i(-1)^{\alpha M}}{\pi(t - \alpha T_A - \bar{\tau}_i)/T_A} \tag{3}$$

where

$$a_i = \frac{1}{\prod_{k=0,k\neq i}^{M-1} \sin(\pi(\bar{\tau}_i - \bar{\tau}_k)/T_A)}, \tag{4}$$

$$\gamma(t) = \prod_{k=0}^{M-1} \sin(\pi(t - \bar{\tau}_k)/T_A) \tag{5}$$

with $T_A = MT_s$ denoting the sampling period of a single converter and $\bar{\tau}_k = kT_s + \tau_k$.

Under the assumption that $\tau \ll T_s$, this reconstruction equation (3) reduces at times $t = nT$ to

$$\hat{x}[n] \quad \approx \quad y[n] - \frac{\tau_i}{T_s}(h * y)[n] \qquad n \pmod{M} = i \tag{6}$$

where

$$h[n] = \begin{cases} 0 & n = 0 \\ \frac{(-1)^n}{n} & \text{otherwise} \end{cases} \tag{7}$$

is a discrete-time derivative filter and $\tau_0 = 0$ as specified previously. This result is derived in the Appendix. The linearity of the τ_i in (6) will be useful for our algorithm development. We let vector $\boldsymbol{\tau}$ represent the unknown timing skews

$$\boldsymbol{\tau} = [\tau_1 \quad \tau_2 \quad \ldots \quad \tau_{M-1}]^T \tag{8}$$

and signal $\hat{x}[n; \boldsymbol{\tau}]$ denote the reconstruction parameterized by $\boldsymbol{\tau}$.

In our formulation, we develop an algorithm that seeks to find parameter values that give a signal reconstruction with no out-of-band energy in the absence of quantization noise and modeling error. We focus on the class of nontrivial input signals for which nonuniform sampling yields aliased content in the frequency band $\omega_c < \omega < \pi$. It is straightforward to establish that for these signals, the out-of-band energy is zero if and only if the parameter values so determined are the correct ones. To see this, we define the error signal $e[n]$ for an arbitrary estimate $\hat{\boldsymbol{\tau}}$ as follows

$$e[n] \quad = \quad \hat{x}[n; \boldsymbol{\tau}] - \hat{x}[n; \hat{\boldsymbol{\tau}}] \tag{9}$$

$$= \begin{cases} 0 & n \pmod{M} = 0 \\ \frac{\hat{\tau}_i - \tau_i}{T_s}(h * y)[n] & n \pmod{M} = i. \end{cases} \tag{10}$$

The reconstruction $\hat{x}[n; \hat{\boldsymbol{\tau}}]$ is bandlimited when $e[n]$ is bandlimited, which is only true if $\hat{\boldsymbol{\tau}} = \boldsymbol{\tau}$. The calibration methods presented also work for a larger class of signals, where the aliased content due to nonuniform sampling appears in other bands of the spectrum; in this case, the algorithms can be redefined with small modifications made to the filters.

IV. Least Squares Method

In this section, we present a least squares formulation for estimating the unknown converter timing skews. We first rewrite (6) in matrix form as

$$\hat{\boldsymbol{x}} \quad \approx \quad \mathbf{y} - \sum_{i=1}^{M-1} \frac{\tau_i}{T_s} \mathbf{D}_i \mathbf{H} \mathbf{y} \tag{11}$$

with vectors representing the received ADC signal and the estimated signal

$$\mathbf{y} \quad = \quad [y[0] \quad y[1] \quad \ldots \quad y[N-1]]^T \tag{12}$$

$$\hat{\boldsymbol{x}} \quad = \quad [\hat{x}[0] \quad \hat{x}[1] \quad \ldots \quad \hat{x}[N-1]]^T, \tag{13}$$

and $N \times N$ Toeplitz matrix $\mathbf{H}$ representing the h filter, where $\mathbf{H}(j,k) = h[j-k]$. The $\mathbf{D}_i$ are $N \times N$ matrices which select the entries from the ith ADC channel,

$$\mathbf{D}_i(j,k) = \begin{cases} 1 & j = k, \; j - 1 \pmod{M} = i \\ 0 & \text{otherwise} \end{cases}. \tag{14}$$

For the sake of analysis, tail effects of filtering are ignored, although their treatment is more carefully handled during simulation.

As shown previously, for our class of input signals, only the true timing skews $\boldsymbol{\tau}$ will yield a signal $\hat{\boldsymbol{x}}$ which is bandlimited. Thus, for an accurate reconstruction, $\hat{\boldsymbol{x}} = \mathbf{L}\hat{\boldsymbol{x}}$, where $\mathbf{L}$ is a matrix implementing a low-pass filter bandlimited to ω_c. In the absence of noise, the timing skews $\boldsymbol{\tau}$ can be computed by solving the linear set of equations $(\mathbf{L} - \mathbf{I})\hat{\boldsymbol{x}} = \mathbf{0}$.

In practice, no solution exists due to modeling error in the approximation (6) and quantization error; therefore, the optimization is formulated as a least squares problem which computes the timing skews $\hat{\boldsymbol{\tau}}$ that minimize the out-of-band energy in $\hat{\boldsymbol{x}}$

$$\hat{\boldsymbol{\tau}} \quad = \quad \arg\min_{\boldsymbol{\tau}} ||(\mathbf{L} - \mathbf{I})\hat{\boldsymbol{x}}|| \tag{15}$$

$$= \quad \arg\min_{\boldsymbol{\tau}} ||\boldsymbol{\gamma} - \mathbf{R}\boldsymbol{\tau}|| \tag{16}$$

where

$$\mathbf{R} = \begin{bmatrix} | & | & & | \\ \mathbf{r}_1 & \mathbf{r}_2 & \ldots & \mathbf{r}_{M-1} \\ | & | & & | \end{bmatrix}, \quad \mathbf{r}_i = (\mathbf{L} - \mathbf{I})\mathbf{D}_i\mathbf{H}\mathbf{y} \tag{17}$$

and

$$\boldsymbol{\gamma} = (\mathbf{L} - \mathbf{I})\mathbf{y}. \tag{18}$$

The solution to the overconstrained least squares estimation problem is given by

$$\hat{\boldsymbol{\tau}} = \left(\mathbf{R}^T\mathbf{R}\right)^{-1} \mathbf{R}^T \boldsymbol{\gamma} \tag{19}$$

where the inversion of $\mathbf{R}^T\mathbf{R}$ is possible because for $N \gg M$ and nontrivial $\mathbf{y}$, $\mathbf{R}$ has full column rank. Thus, with $O(MN)$ complexity, the optimal solution $\hat{\boldsymbol{\tau}}$ can be computed. Uniform samples of the input signal can then be recovered from the timing skew estimates via (6). In a geometric interpretation, the least squares method computes the signal in the convex set $\mathcal{S}_{\mathcal{A}}$ that is closest to the convex set $\mathcal{S}_{\mathcal{B}}$, where $\mathcal{S}_{\mathcal{A}}$ represents the set of signals $\hat{x}[n; \boldsymbol{\tau}]$ spanned by $\boldsymbol{\tau}$ and $\mathcal{S}_{\mathcal{B}}$ represents the set of signals bandlimited to ω_c.

A. Relinearization

For values of τ_i/T_s which are not sufficiently close to zero, the approximation given by (6) may only provide a coarse reconstruction of the original signal because it relies on a Taylor series expansion of (3) around $\boldsymbol{\tau} = \mathbf{0}$. In this case, we present an iterative method for improving the accuracy of the initial least squares estimate. Similar to Newton's method, we perform successive approximations by first computing the least squares estimate $\hat{\boldsymbol{\tau}}$ and then computing the first order Taylor series approximation of $x[n]$ around the point $\boldsymbol{\tau} = \hat{\boldsymbol{\tau}}$

$$\hat{x}[n] \quad \approx \quad x[n]\Big|_{\boldsymbol{\tau}=\hat{\boldsymbol{\tau}}} + \sum_{i=1}^{M-1} \frac{\partial x[n]}{\partial \tau_i}\Big|_{\boldsymbol{\tau}=\hat{\boldsymbol{\tau}}} \tau_i. \tag{20}$$

From the updated reconstruction formula (20), it is possible to formulate a new least squares problem whose solution is another estimate of $\boldsymbol{\tau}$. With increasingly accurate estimates $\hat{\boldsymbol{\tau}}$, the local approximation of $x[n]$ can improve and allow for better estimation. Its benefits are presented in Section V.

B. Gains

In the general calibration of time-interleaved analog-to-digital converters, nonuniform gains also exist among the constituent converters. In this setup, the output of the ith ADC is modeled as

$$y_i[n] = g_i x(MT_s n + \tau_i) + w_i[n]. \tag{21}$$

where the g_i are unknown gains. Although the gains vary among the converters, we assume that each gain is within 5% of unity. For high resolution converters, one can conveniently compensate for the system gains without excessive noise enhancement by multiplying each ADC output $y_i[n]$ by $1/g_i$. Without loss of generality, we set $g_0 = 1$.

By folding the gain recovery into the reconstruction equation (3), we can compute the Taylor series approximation around the point $\boldsymbol{\tau} = \mathbf{0}, \mathbf{g} = \mathbf{1}$, where $\mathbf{g} = [g_1 \;\; g_2 \;\; \ldots \;\; g_{M-1}]$ and $\mathbf{0}$, $\mathbf{1}$ are vectors of zeros and ones respectively. From the first order approximation, we can setup a similar least squares problem which now includes gains in the vector of unknown parameters

$$\hat{\boldsymbol{\Theta}} \quad = \quad \arg\min_{\boldsymbol{\Theta}} ||\gamma - \mathbf{R}\boldsymbol{\Theta}|| \tag{22}$$

where

$$\boldsymbol{\Theta} \quad = \quad \begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{1/g} \end{bmatrix} \tag{23}$$

$$\mathbf{R} \quad = \quad \begin{bmatrix} | & & | & | & & | \\ \mathbf{r}_1 & \ldots & \mathbf{r}_{M-1} & \mathbf{s}_1 & \ldots & \mathbf{s}_{M-1} \\ | & & | & | & & | \end{bmatrix} \tag{24}$$

$$\mathbf{r}_i \quad = \quad (\mathbf{L} - \mathbf{I})\mathbf{D}_i\mathbf{H}\mathbf{y} \tag{25}$$

$$\mathbf{s}_i \quad = \quad -(\mathbf{L} - \mathbf{I})\mathbf{D}_i\mathbf{y} \tag{26}$$

$$\gamma \quad = \quad (\mathbf{L} - \mathbf{I})\mathbf{D}_0\mathbf{y} \tag{27}$$

and for notational compactness $\mathbf{1/g}$ is used to denote a vector containing the inverses of the gains.

V. Results

In this section, we numerically evaluate the performance characteristics of the blind calibration methods. We compare the effective number of bits for the reconstruction without calibration $y[n]$, to the reconstruction with calibration, i.e., $\hat{x}[n]$.

To measure effective bits, we first compute the signal-to-noise ratio (SNR) of the recovered signal $\hat{x}[n]$

$$\text{SNR}_{\hat{x}} = 10\log_{10}\frac{\sum x[n]^2}{\sum (x[n] - \hat{x}[n])^2}. \tag{28}$$

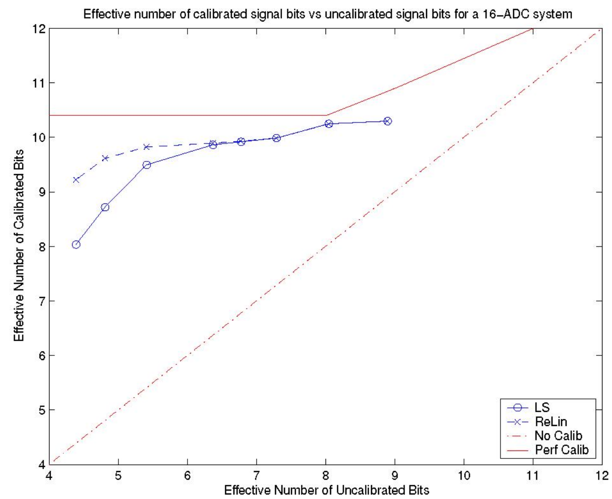

Fig. 2. Effective number of calibrated bits vs. uncalibrated bits for a 12-bit 16-ADC system with unknown timing skews. Performance is measured using the least squares (LS) estimate and the relinearization (ReLin) estimate for a bandlimited Gaussian input oversampled by 33%.

The uncalibrated signal SNR can be calculated in a similar fashion. The effective SNR of a signal is then related to the number of effective bits via $B = (\text{SNR} - 1.76)/6.02$. In the tests below, the converter quantizes the input at 12-bit resolution which generates the noise $w_i[n]$ in (1); the performance is measured through the increase in effective bits between the uncalibrated and calibrated signals. We now discuss the tradeoffs in performance between the amount of excess bandwidth, block size, number of converters and input SNR.

For our simulations, we randomly select the $M - 1$ converter timing skews independently using a uniform distribution. Increasing the range of this distribution yields a lower number of effective bits in the uncalibrated signal $y[n]$. In Figure 2, we plot the performance of the calibration methods by showing the effective bits of the calibrated signal vs. uncalibrated signal in a system with 16 ADCs. Performance is shown for tests with a single least squares estimate and tests with five iterations of the relinearization method. In this case, the gains are uniform among all converters. The tests are performed using bandlimited Gaussian noise as input, with block sizes of 2^{15} samples and a factor of 33% oversampling. For additional precision, the final estimate $\hat{x}[n]$ is produced using the time-skew estimates in the ideal reconstruction formula (3) rather than the estimated reconstruction formula, which provides ~ 0.3 more effective bits. The upper bound (solid curve) shows simulated recovery performance when the true timing skew values are used in the ideal reconstruction formula; out-of-band energy in the input, quantization noise, and numerical precision errors limit the performance below 12 effective bits. The lower bound (dashed line) plots the performance when no recovery is performed, i.e. effective output bits equals effective input bits.

As the timing skew decreases (increasing uncalibrated bits), the recovery algorithm yields a higher number of effective output bits. The addition of the relinearization method significantly increases performance in the cases with high numbers of ADCs and large timing skews. For the 16-ADC system with ~ 5 effective uncalibrated bits, the relinearization method achieves an output performance of 9.7 bits ($\sim 0.03\%$ error in time skew estimates), which is 0.9 bits better than the least squares method. Although the performance of the relinearization estimate generally increased with more iterations, some tests showed a decrease in performance, possibly due to a nonlinear structure around the point of approximation.

For a fixed oversampling rate, tests with smaller numbers of converters showed better performance. With more converters, the number of timing skews increased causing additional sources of estimation error. In general, the decrease in performance for larger numbers of ADCs is more apparent at lower levels of uncalibrated bits. At a level of six effective uncalibrated bits, the relinearization method for a 2-ADC system achieved 0.6 bits better than a 16-ADC system.

Tradeoffs in performance were also measured for varying input block sizes and amounts of oversampling. After a baseline amount of oversampling (15%) and block size (2^{11} samples/ADC), varying these parameters had marginal effects ($\sim$ 0.4 bits) on the effective bits in the calibrated signal. However, the convergence speed of the algorithm was highly dependent on the oversampling factor; with more oversampling, the least squares estimate provided higher accuracy and relinearization only made small improvements.

Additional tests were performed for signal calibration in time-interleaved systems that contain both unknown timing skews and unknown gains. The $M-1$ gains are chosen independently using a uniform distribution. Although the initial least squares estimate was often of poor quality, the relinearization technique achieved ~ 10 bit performance for $M < 32$ converters, which is similar to the performance of tests when only timing skews were unknown. When gains are present, the number of uncalibrated bits can be both a function of the range of the timing skew and the range of the gains, i.e. a 4-bit signal can be produced by high timing skews or high gains. Signal recovery performance is dependent on whether the gain range or timing skew range is higher. When a larger gain range causes the decrease in uncalibrated bits, the calibration methods are slightly more effective than when a larger skew range causes the decrease in bits.

VI. Conclusion

We have presented an algorithm for blind calibration of high-resolution time-interleaved analog-to-digital converters. Using a least squares formulation, the algorithm produces accurate estimates of the time-skew parameters and reconstructs samples of the input on a uniform grid. We exploit the linearity of time-skew parameters in high-resolution converters where skews are small relative to the sampling period. The algorithm shows promising performance and its ability to scale with linear complexity makes it an attractive solution for large numbers of converters. With the addition of unknown gains and relinearization techniques, the method is robust towards handling a broader class of time-interleaved analog-to-digital converter systems.

The upper bound on the performance of the least squares and relinearization algorithms needs further investigation. In particular, it is unclear whether accurate convergence of the estimates is guaranteed in the absence of numerical precision errors or algorithmic approximations, especially in the cases of large numbers of converters $M > 32$. Furthermore, the effect on performance of input signal out-of-band energy also needs to be evaluated.

Appendix
Derivation of the Reconstruction Equation

We now construct a derivation of the reconstruction approximation (6). For the M-ADC system, the first order Taylor series approximation of (3) at times $t = nT$ around the point $\boldsymbol{\tau} = \mathbf{0}$ is given by

$$\hat{x}[n] \approx x[n]\Big|_{\boldsymbol{\tau}=\mathbf{0}} + \sum_{i=1}^{M-1} \frac{\partial x[n]}{\partial \tau_i}\Big|_{\boldsymbol{\tau}=\mathbf{0}} \tau_i \quad (29)$$

$$= y[n] - \frac{\tau_i}{T_s}(h * y)[n] \qquad n \pmod M = i \quad (30)$$

where

$$h[n] = \begin{cases} 0 & n = 0 \\ \frac{(-1)^n}{n} & \text{otherwise} \end{cases}. \quad (31)$$

In high resolution converters, the approximation (29) is valid due to the fact that the τ_i/T_s are small in magnitude.

There is a nice interpretation of the preceding result. We now present an alternate derivation which gives the intuition that in our reconstruction formula, we correct each sample by treating all the other samples around it as being sampled uniformly (although this is not actually the case). To show this, we perform a coarse approximation from the interpolation equation $x(t) = \sum x[\alpha]\text{sinc}(t - \alpha T_s)$. For $n \pmod M = i$:

$$y[n] = x(nT_s + \tau_i) = \sum_{\alpha} x[\alpha]\text{sinc}\left((n-\alpha)T_s + \tau_i\right) \quad (32)$$

$$x[n] = \frac{1}{\text{sinc}(\tau_i)}\left(y[n] - \sum_{\alpha \neq n} x[\alpha]\text{sinc}\left((n-\alpha)T_s + \tau_i\right)\right) \quad (33)$$

$$\approx \frac{1}{\text{sinc}(\tau_i)}\left(y[n] - \sum_{\alpha \neq n} y[\alpha]\text{sinc}\left((n-\alpha)T_s + \tau_i\right)\right) \quad (34)$$

$$\approx y[n] - \frac{\tau_i}{T_s}(h * y)[n] \quad (35)$$

where the second approximation is obtained by the first order Taylor series expansion of the sinc function

$$\text{sinc}(nT_s + \tau_i) \approx \begin{cases} 1 & \text{n=0} \\ \frac{(-1)^n}{n}\frac{\tau_i}{T_s} & \text{otherwise} \end{cases}. \quad (36)$$

Thus, in the correction of a single sample, the derived reconstruction method (6) is equivalent to making the approximation that all the neighbors of the sample are on a uniform grid.

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