On the Coding-Spreading Tradeoff and Intra-Cell Frequency Planning in Uplink CDMA Systems

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***Abstract*— We study potential gains in the spectral efficiency of multi-cell uplink CDMA systems that accrue from assigning users in different parts of a cell to different frequency bands. We develop a criterion that enables a fair comparison with a conventional (non-partitioned) system. Two design scenarios are considered: (i) each user requires a fixed data rate, (ii) each user has a fixed SNR. We show that in both scenarios there is a gain in spectral efficiency—regardless of the particular partitioning strategy employed—if an MMSE receiver is used. There is no gain if a decorrelator or a matched filter receiver is used. We validate our results through numerical simulations and provide an intuitive explanation based on the coding-spreading tradeoff.**

I. Introduction

Spread spectrum CDMA is an attractive technology for multi-access communications due to features such as robustness to multipath fading and soft handoff capabilities. A question of practical importance is how to design such systems to maximize system capacity. Improved frequency planning is one design resource available for tuning. Conventional CDMA systems reuse the entire spectrum for all users in all cells (Figure 1(a)). While it is intuitively clear that using different portions of the spectrum in different cells (Figure 1(b)) degrades capacity, we instead consider the less usual case of partitioning the spectrum into multiple groups within each cell (Figure 1(c)).

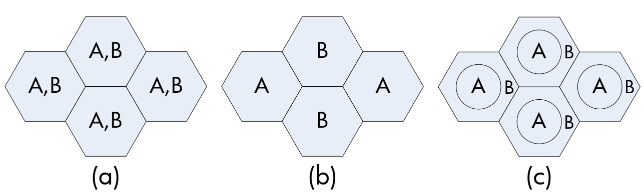

Fig. 1. Cellular reuse: (a) conventional CDMA system with reuse 1, (b) ill-advised CDMA system with reuse 2, (c) reuse 1 with intra-cell partitioning

Why is such intra-cell partitioning beneficial? In a single cell scenario, it's not: with perfect uplink power control users in groups A and B are received with the same power at the base station. The groups "look" the same to a receiver and there is no advantage to partitioning. But with multiple cells the picture changes: cell-edge users cause more interference to neighboring cells than do cell-interior ones. By partitioning users into two pools—those that cause high intercell interference and those that cause low intercell interference—we can separately tune the system parameters (e.g., the coding-spreading tradeoff) for each pool and improve spectral efficiency.

An intra-cell frequency partitioning scheme for *downlink* CDMA is proposed in [4]. The main idea in this paper is that novel frequency planning schemes can be designed for downlink CDMA that provide improved interference suppression. However their schemes do not provide interference suppression in the uplink scenario. We are not aware of prior work on gains in spectral efficiency from frequency planning in uplink CDMA.

In order to address the problem of uplink CDMA, we use the information theoretic tools developed in [1], [2]. In [1], the spectral efficiency for a single cell system is derived in the limit as the number of users (K) and the spreading factor (N) approach infinity, with the ratio K/N being constant. Explicit expressions are derived for different multi-user receivers, including the matched filter, the decorrelator, and the linear MMSE receiver. Interpretation of this analysis in terms of the coding-spreading tradeoff in CDMA systems is provided in [3]. The authors fix the per-user data rate (R_b bits/second) and derive a relationship between the spectral efficiency of the system and the code rate. They show that, while for a matched filter the coding-spreading tradeoff favors coding, the linear MMSE receiver has a non-trivial coding-spreading tradeoff. The paper also studies a simple model for the multi-cell scenario for conventional systems.

In this paper we study the potential gains in spectral efficiency of *uplink* multi-cellular CDMA systems from intra-cell frequency partitioning. We introduce a fairness constraint to enable comparison between a partitioned system and a conventional one. With this constraint, the gains depend on the structure of the receiver. Our main result is that frequency partitioning increases the spectral efficiency of CDMA systems that employ a linear MMSE receiver. The result follows from the convexity of a particular capacity expression, and it holds for arbitrary partitions of users including the particular near/far partition shown in Figure 1(c). In contrast, if a matched filter or a decorrelator is used, there are no gains from frequency partitioning. We interpret these results in terms of the coding-spreading tradeoff. We also provide simulation results using simple interference models to quantify the gains resulting from frequency partitioning.

The paper is organized as follows. In Section II we develop the system model and Section III we detail the design of fixed data rate and fixed SNR systems. In Section IV we discuss the known results for conventional systems. In Section V we develop a fairness criterion for frequency partitioned systems and in Section VI we study the spectral efficiency of such systems.

II. System Model

We consider a CDMA system with W Hz of available spectrum. Each user communicates with the nearest base station with perfect power control and is received with power P. The transmitter for each user consists of an ideal AWGN encoder which produces coded symbols at R bits/symbol which are then spread using a pseudorandom spreading sequence at a rate of N chips per symbol as shown in Figure 2. We consider a symmetric cellular model so that the received signal at the base station is a superposition of signals from the in-cell users and the interfering users from first tier. The discrete time signal at the receiver may then be expressed as

$$\mathbf{Y} = \underbrace{\sum_{i=1}^{K} x_i \mathbf{s_i}}_{\text{in-cell}} + \underbrace{\sum_{j=1}^{6}\sum_{i=1}^{K} h_{ij} x_{ij} \mathbf{s_{ij}}}_{\text{first-tier}} + \mathbf{w}. \tag{1}$$

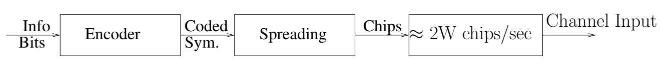

Fig. 2. Structure of transmitter for each user

We assume the coded symbols x_i and x_{ij} are mutually independent real $N(0, P)$ random variables. The components of the spreading sequences $\mathbf{s_i}$ and $\mathbf{s_{ij}}$ are randomly and independently chosen from $\{-1, 1\}^N$. The base station knows the spreading sequence of each in-cell user and the users in the first tier.[1] The noise vector is $\mathbf{w} \sim N(0, \sigma^2 I)$. The coefficients h_{ij} are the attenuations experienced by the out-of-cell user signals. In the limit as the number of users $K \to \infty$, the empirical distribution of these attenuation factors converges to some distribution $F(h)$ [2]. Furthermore, from symmetry this fixed distribution is the same for each adjacent cell. The overall attenuation distribution $F_T(h)$ can be written as a sum of the in-cell and adjacent cell distributions:

$$F_T(h) = \frac{1}{7}\underbrace{\delta(h-1)}_{\text{In Cell}} + \frac{6}{7}\underbrace{F(h)}_{\text{First Tier}} \tag{2}$$

The attenuation distribution $F(h)$ can also be interpreted as the adjacent cell interference distribution, as it controls the interference term in (1). When in Section V we partition users into two groups, each will have a different distribution $F_1(h)$ and $F_2(h)$. The inner group will have a more favorable distribution than the outer one. Intuitively, the greater the disparity between F_1 and F_2 the greater the potential gains one can expect from partitioning.

[1]This assumption is often used in multicell CDMA, e.g., [3], [5].

TABLE I
Design Parameters in CDMA system

Scenario	Objective	Fixed Params	Design Params
I	max K	R_b, γ_b	R (or N)
II	max $(K \times R_b)$	N, γ_s	R (or K)

III. System Design & Coding-Spreading Tradeoff

In a CDMA system design, some of the parameters are typically viewed as fixed. These include the bandwidth W and received power P of in-cell users. Assuming the users spread over the entire available spectrum, this also fixes the chip rate $T_c = 1/2W$ and the energy per chip $E_c = PT_c$. Adjustable parameters in the system typically include the spreading factor N (chips/symbol), the code rate R (bits/symbol), the number of users K, and the data rate per user R_b (bits/sec). The spectral efficiency of such a system is defined as $C = \frac{KR}{N}$ bits/chip [1]. In what follows, we consider two practical design scenarios and show how the optimizing C is natural in both.

A. Fixed Data Rate System

In Scenario I we fix the data rate R_b (bits/second) for each user in the network. This also fixes the energy per bit $E_b = P/R_b$ and equivalently $E_b/N_0 = \gamma_b$. The objective is to maximize the number of users K in each cell by optimizing over N (or R). Note that by fixing R_b we fix also the ratio $N/R = 2W/R_b$. Thus the spectral efficiency is proportional to the number of users in the system.

B. Fixed SNR System

In Scenario II we fix the input SNR $\gamma_s = NE_c/N_0$ in the system. This is equivalent to fixing the spreading gain N and hence the symbol duration. We optimize over the number of users in each cell (or equivalently over the code rate R) to maximize the total throughput KR_b in each cell. This quantity is proportional to the spectral efficiency of the system.

Table I summarizes the parameters of the two scenarios. In both cases the objective is equivalent to maximizing spectral efficiency by adjusting the code rate R. In Scenario I we fix $E_b/N_0 = \gamma_b$ whereas in Scenario II we fix the input SNR γ_s for each user. In Scenario I the optimizing R can be directly related to the coding-spreading tradeoff [3]. If the optimal value of R is small then the system favors bandwidth expansion via coding. If R is large then the system favors bandwidth expansion via linear spreading. In Scenario II the interpretation is slightly different: small R corresponds to many users with a low rate while large R corresponds to fewer users with a high rate[2].

IV. Performance of Conventional Systems

In this section we briefly review the analysis of spectral efficiency for conventional CDMA systems and provide simulations and discussions which lead to useful insights for our later results. We will find it convenient to express spectral efficiency as a function of the effective SINR (signal to

[2]Scenario II is directly analogous to a smart antenna design problem. Scenario I has no such interpretation.

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interference plus noise ratio) for each user. This quantity is directly related to the code rate by $R = 1/2\log(1+\text{SINR})$. The analysis assumes large system limit ($K, N \to \infty$, K/N = α). The base station has a linear multiuser receiver that produces K effective single user channels. If β is the output SINR of these channels, the spectral efficiency is[3]

$$C = \max_{\beta} \alpha(\beta)\log(1+\beta), \tag{3}$$

where $\alpha(\beta)$ is the number of users per degree of freedom for a target output SINR (β), as calculated in [2]. We specialize this result to Scenario II (fixed SNR γ_s) and the interference model (2). The appropriate $\alpha(\beta)$ for the matched filter (mf), decorrelator (dec), and linear MMSE (mmse) receivers are as follows:

$$\alpha_{\text{mf}} = \frac{\frac{1}{\beta_{\text{mf}}} - \frac{1}{\gamma_s}}{1+6E[H]} \tag{4}$$

$$\alpha_{\text{dec}} = \frac{1}{7}\left(1 - \frac{\beta_{\text{dc}}}{\gamma_s}\right) \tag{5}$$

$$\alpha_{\text{mmse}} = \left\{ \frac{\frac{1}{\beta_{\text{mmse}}} - \frac{1}{\gamma_s}}{\frac{1}{1+\beta_{\text{mmse}}} + 6\int_0^1 \frac{1}{\frac{1}{h}+\beta_{\text{mmse}}} F(h)dh} \right\} \tag{6}$$

Similar expressions can be derived for Scenario I using the relation $\gamma_b = \gamma_s \log(1+\beta)$. Substituting in (3) and optimizing over β we obtain the spectral efficiency for each receiver.

A. Simulations and Discussion

The analytical expressions are difficult to compute and depend on the choice of $F(h)$ in (2). Along the lines of [3], [5], we evaluate the spectral efficiency for an extremely simple interference model $F(h) = \delta(h - 1/12)$. This model implies that each out-of-cell interferer is received at a power which is 1/12 of the in-cell users' strength. While the model is not realistic it provides useful intuition for the performance of different receivers. The spectral efficiency can be numerically optimized and is plotted in Figure 3.

In Figure 3, the top graph is for Scenario II (fixed γ_s) and the lower graph is for Scenario I (fixed γ_b). The main observation is that in an optimized system at low γ_s (or γ_b) the MMSE receiver approaches the performance of a matched filter whereas for high γ_s (or γ_b) it approaches that of a decorrelator.

Figure 4 further details this phenomenon. The upper plot shows the achievable throughput as a function of the code rate for three different values of γ_b (Scenario I). We see that this function is bimodal in the code rate. The first peak is attributed to a matched filter-like behavior whereas the second peak is attributed to a decorrelator-like behavior. As γ_b increases, the value of the second peak increases and the MMSE performance approximates that of the decorrelator. For $\gamma_b \approx 24$ dB, the two peaks are the same and so we observe a sharp transition in the optimal code rate in the lower plot. This plot shows that for low E_b/N_0 ($\gamma_b \leq 5$ dB), the optimal

[3]This definition is slightly different from [1]. They do not maximize over β, but consider $C(\beta)$ as spectral efficiency.

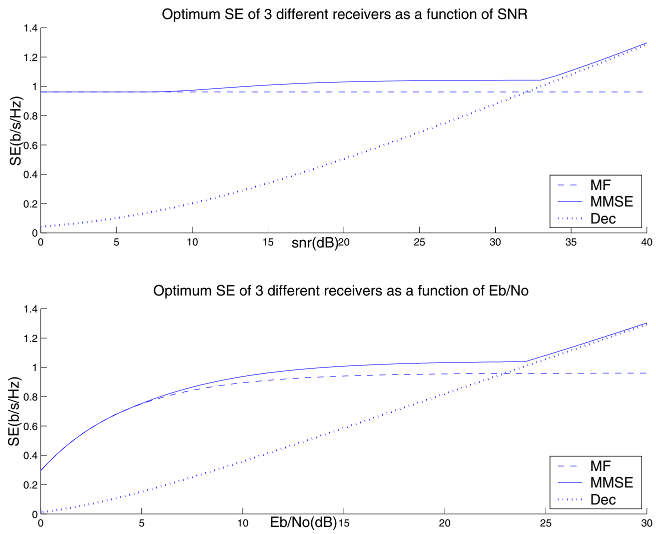

Fig. 3. Spectral efficiency of different multiuser receivers

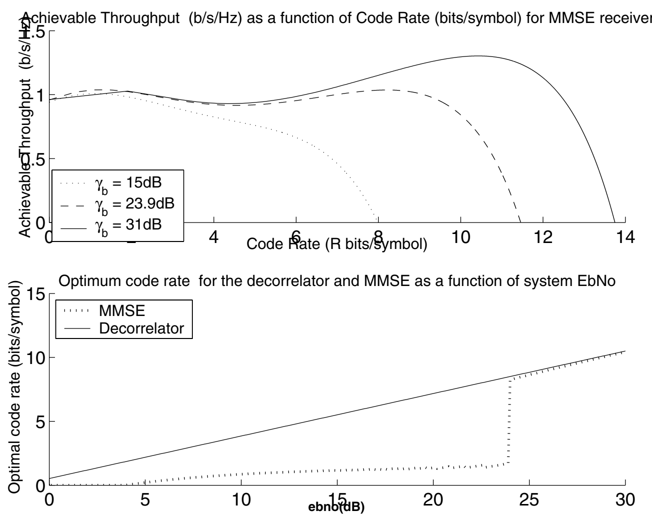

Fig. 4. Performance of MMSE receiver

code rate $R_{\text{opt}} \approx 0$ while for high E_b/N_0 ($\gamma_b \geq 25$ dB), the optimal operating point is essentially the same as that of the decorrelator. Note that the optimal code rate for matched filter is always 0 [3]. For intermediate values of γ_b the optimal operating code rate is non-zero and not equal to that of the decorrelator. In this regime MMSE performs better than the matched filter and the decorrelator (Figure 3) and has a non-trivial coding-spreading tradeoff.

V. Fairness in Frequency Partitioning

We describe an intra-cell frequency partitioning scheme that divides the users into two service groups, each group assigned disjoint spectra. To simplify the exposition we consider a large population of potential users such that half the users are assigned to Group 1 and the other half to Group 2. A randomly selected user is equally likely to be in either group. Users in each group employ perfect power control and are received with power P. However, the users differ in the interference they cause to (and also experience from) adjacent cells. This effect is modeled by the different attenuation distributions (cf. (2)) $F_1(h)$ and $F_2(h)$ of the two groups. For comparison, a conventional system without intra-cell partitioning experiences the average attenuation distribution $F(h) = (F_1(h) + F_2(h))/2$.

We now consider the effect of partitioning the bandwidth W into two disjoint bands W_1 and W_2. The problem formulation for Scenario I is unchanged: we hold the data rate (equivalently γ_b) constant for all users in both groups. In Scenario II, however, the variable bandwidths W_1 and W_2 cause the chip durations T_c and hence the spreading factors N_1 and N_2 to vary between the groups even as the SNR is held constant across the groups. (See Table I.)

From (3)–(6) it is clear that the spectral efficiency is a function of the attenuation distribution $F(\cdot)$ and the SNR γ_s (or the E_b/N_0 γ_b). Even though the two groups have same γ_s (or γ_b), they have different spectral efficiencies C_1 and C_2 because the attenuation distributions are different. Assume $C_1 > C_2$. If the objective were simply to maximize the overall spectral efficiency, one would allocate all bandwidth to Group 1. However, such an allocation isn't fair because half the users would receive no service. To avoid this problem we adopt a natural criterion of fairness that requires the total throughput of the two groups to be equal.

Definition 1 (Fair Partitioning): A fair partition of bandwidth W between two equiprobable groups with spectral efficiency C_1 and C_2 satisfies $C_1 W_1 = C_2 W_2$.

For the fixed rate scenario, this reduces to requiring that the total number of users in each group is the same. Our design is fair is the sense that no matter which group the user is in, he/she is equally likely to get served, just like the conventional system which does not partition users. For the fixed SNR case, fairness requires $K_1 R_1 = K_2 R_2$.

With this fairness criterion, the overall spectral efficiency C_{fair} of the system and the resulting gain Ω satisfy

$$\frac{1}{C_{\text{fair}}} = \frac{1/2}{C_1} + \frac{1/2}{C_2}, \quad (7)$$

$$\Omega = C_{\text{fair}}/C. \quad (8)$$

The reciprocal form of (7) motivates the convexity result for $1/C$ (rather than for C) derived in Section VI.

Note that the above definition for fairness can easily be extended to the case where groups are not equiprobable.

VI. Gains from Frequency Partitioning

We now prove that intra-cell frequency partitioning under the fairness criterion described in the previous section increases spectral efficiency. We prove the result for the fixed SNR case; the case of fixed E_b/N_0 is analogous.

For an MMSE receiver system, operating at an SINR of β_{MMSE}, attenuation distribution $F(\cdot)$, and SNR γ_s, we define the spectral efficiency $C(\beta_{\text{MMSE}}, \gamma_s, F)$ by substituting (6) in (3) without optimizing over β_{MMSE}. The spectral efficiency $C(\gamma_s, F)$ follows by optimizing this achievable rate over all $\beta_{\text{MMSE}} \geq 0$. Using these definitions, the advantage of intra-cell partitioning may be expressed in terms of convexity.

Theorem 1: For fixed γ_s, $\frac{1}{C(\gamma_s,F)}$ is convex ($\cap$) in F.

Proof: Combining (6) and (3) we have

$$C(\beta_{\text{MMSE}}, \gamma_s, F) = \frac{\left(\frac{1}{\beta_{\text{MMSE}}} - \frac{1}{\gamma_s}\right) \log_2(1 + \beta_{\text{MMSE}})}{\frac{1}{1+\beta_{\text{MMSE}}} + 6 \int_0^1 \frac{1}{\frac{1}{h} + \beta_{\text{MMSE}}} F(h) dh}. \quad (9)$$

Thus $1/C(\beta_{\text{MMSE}}, \gamma_s, F)$ is linear in F. Let

$$F(\cdot) = \delta F_1(\cdot) + (1 - \delta) F_2(\cdot)/ \quad (10)$$

be a convex combination of F_1 and F_2 for some $\delta \in [0, 1]$. Then

$$\begin{aligned}
\frac{1}{C} &= \min_{\beta_{\text{MMSE}}} \left\{ \frac{1}{C(\beta_{\text{MMSE}}, \gamma_s, F)} \right\} \\
&\overset{a}{=} \min_{\beta_{\text{MMSE}}} \left\{ \frac{\delta}{C(\beta_{\text{MMSE}}, \gamma_s, F_1)} + \frac{1-\delta}{C(\beta_{\text{MMSE}}, \gamma_s, F_2)} \right\} \\
&\overset{b}{\geq} \min_{\beta_{\text{MMSE}}} \left\{ \frac{\delta}{C(\beta_{\text{MMSE}}, \gamma_s, F_1)} \right\} + \min_{\beta_{\text{MMSE}}} \left\{ \frac{1-\delta}{C(\beta_{\text{MMSE}}, \gamma_s, F_2)} \right\} \\
&= \frac{\delta}{C_1} + \frac{1-\delta}{C_2}.
\end{aligned}$$

Here (a) follows by substituting (10) in (9), while (b) is obvious. ■

The frequency partitioning scheme described in the previous section is the special case $\delta = 1/2$. It follows from (7) that $C_{\text{fair}} > C$. More generally, if the two groups are not equiprobable, then the relation between F, F_1, and F_2 is given by (10) and our fairness criterion is $\delta C_1 W_1 = (1 - \delta) C_2 W_2$. The above theorem shows that any such fair partitioning improves (or at least does not harm) spectral efficiency.

The intuition behind the result is that for the MMSE receiver, the optimal operating SINR is a function of the attenuation distribution. While the conventional system forces both groups to operate at a single SINR, the partitioned system allows the two groups to operate at different SINRs (equivalently different coding-spreading tradeoffs) and this results in higher capacity. We also note from the convexity result that dividing users into more groups yields even higher gains as long as the groups operate at different coding-spreading tradeoffs.

How much gain does partitioning provide? To get some intuition for the nature of the gains, we study the performance when a matched filter and a decorrelator are used.

Corollory 1: Using a decorrelator receiver, $C_{\text{fair}} = C$.

Proof: It follows from (5) and (3) that $C(\beta_{\text{dec}}, \gamma_s, F)$ is independent of F. Thus the optimizing β_{dec} depends only on γ_s. Since the two groups and the conventional system all have the same SNR γ_s, we have $C_1 = C_2 = C$. ■

Corollory 2: Using a matched filter receiver, $C_{\text{fair}} = C$.

Proof: Examining (4) and (3) we see that the optimal β_{mf} is independent of F. Step (b) in Theorem 1 is met with equality and hence $C_{\text{fair}} = C$. ■

The intuition behind these results is that while intra-cell frequency partitioning allows the two groups to operate at different coding-spreading tradeoffs, neither the decorrelator nor the matched filter can take advantage of this feature. One would not expect large gains from the MMSE receiver if its performance degenerates to one of these receivers.

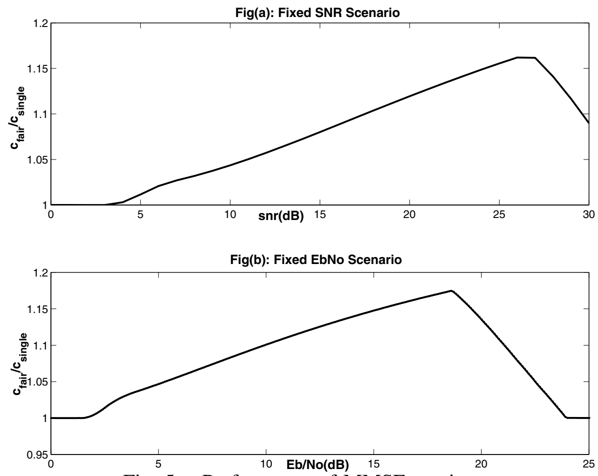

Fig. 5. Performance of MMSE receiver

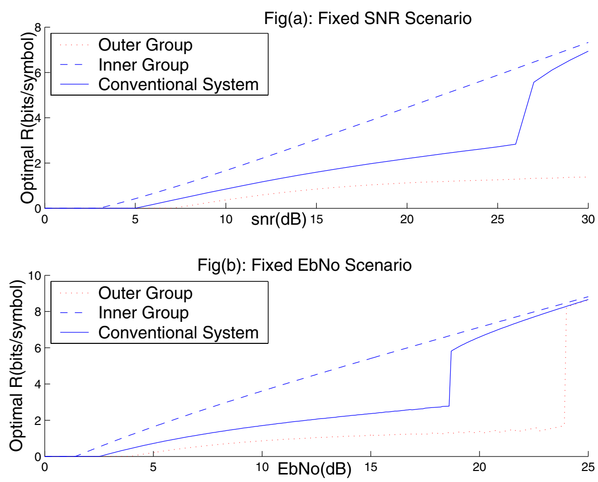

Fig. 6. Optimal code rate for different groups and conventional system

A. Simulation and Discussions

We evaluate the gains achieved through frequency partitioning under a simple interference model. We assume users in Group 1 do not cause any interference to the adjacent cells ($F_1(h) = \delta(h)$), whereas users in Group 2 are all received at a power which is 1/12 of the power of in-cell users ($F_2(h) = \delta(h - 1/12)$). This model is a natural extension of the interference model used in conventional systems described in Section IV-A. It idealizes the scenario where users close to the base station are put in Group 1 and users close to the cell edge are put into Group 2.

Figure 5 shows the gains achieved from equiprobable groups in the case of fixed γ_s (Scenario II) and fixed γ_b (Scenario I). In both the systems the nature of gains is similar. For low values of γ_s or γ_b the MMSE performance is close to that of a matched filter. From Corollary 2, it follows that there is no gain from frequency partitioning in the system. Similarly, for high values of γ_s or γ_b, the gains again diminish as the performance of the MMSE receiver approaches that of the decorrelator and Corollary 1 applies. It is for moderate value of γ_s or γ_b that the the MMSE receiver system gains from frequency partitioning. As shown in Figure 6, the outer and inner groups, in this range, operate at different coding spreading tradeoffs and consequently reasonable gains are seen from partitioning the users. In this range, the outer group experiences high interference and operates at low code rates (closer to a matched filter). The inner group receives interference only from in-cell users operates at higher code rates (closer to a decorrelator). On the other hand the conventional system has to operate at a code rate which compromises these extremes. Figure 5 shows that for $\gamma_b \approx 15$ dB we see a 15% gain over the conventional system.

These calculations are performed for the idealistic model. As the distributions $F_1(h)$ and $F_2(h)$ begin to overlap, the difference between the operating points will decrease and one would expect lower gains. However, our results predict that higher gains could be achieved for any arbitrary partitioning and increase with the number of partitioned groups. It may be possible to achieve higher gains through more clever designs.

VII. Conclusions and Future Work

We developed a model for intra-cell frequency partitioning in uplink CDMA systems under a fairness constraint. We studied the information theoretic spectral efficiency of such systems under the scenarios of fixed data rate and fixed SNR for each user. Our main result is that in either case, the spectral efficiency for MMSE receivers increases from any arbitrary frequency partitioning. We provided intuition for this result in terms of the coding-spreading tradeoff. Numerical analysis was performed for a simple interference model inspired from the idea of segregating users close to the base station and users close to the cell edge. Our simulation results showed moderate gains on the order of 10–20% in the spectral efficiency for intermediate values of SNR or E_b/N_0. It was also shown that for very high or very low SNR or E_b/N_0, such gains are not observed.

One direction for future work is to develop novel frequency partitioning schemes that achieve higher gains. An optimal frequency partitioning design or an upper bound on the achievable gain will provide useful insights in the difficult problem of frequency planning. Another direction is to study the performance gains under more realistic interference models and practical coding schemes. It would also be interesting to analyze this system in the presence of fading.

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