A Capacity Theorem for Cooperative Multicasting in Large Wireless Networks

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Abstract

We consider a scenario in which a sender wishes to broadcast a common message to a large number of receivers in a slowly fading wireless environment. We exploit cooperation between the receivers but impose an average sum power constraint across the network. As the number of receivers $K \to \infty$, there is a critical rate C for the common message such that the outage probability of *every* receiver approaches zero for all rates below C and is bounded away from zero above this rate. We compare our work with previous work on cooperative diversity that focuses on the diversity-multiplexing tradeoff.

1 Introduction

Cooperative diversity has been proposed as a powerful scheme to combat slow fading in wireless networks. Spatially distributed nodes provide an opportunity to create a distributed virtual antenna array, which can provide substantial gains in slow fading environments [1]. Several cooperative diversity protocols have been suggested in [2, 3] and evaluated under the framework of the diversity-multiplexing tradeoff proposed in [4]. Recently, some new cooperative diversity based protocols have been proposed in [5, 6, 7], that achieve better performance in the sense of diversity-multiplexing tradeoff than previously proposed protocols.

Common to all these works is

- There is only one destination receiver and a small number of nodes act as relay terminals.
- To make the performance of the protocols amenable to analysis, a high SNR assumption is imposed. The analysis is done in the framework of the diversity-multiplexing tradeoff.

The present work studies cooperative diversity protocols for multicasting in a slow fading environment but follows a different route, in both the setup and the analysis technique. First, instead of focusing on the limit of high SNR in order to simplify the analysis, we focus on the limit of many users. Second, in addition to studying the single intended receiver case as done in previous works, we also consider the case where all the receivers want to decode the same message from one sender. Such a system is clearly limited by the worst user in the network. Accordingly, we consider the system to be in "outage" if at least one user cannot decode the message.

It is well known that in point-to-point links with slow fading the Shannon capacity is zero (see, e.g., [8]). This follows since for *any* target rate, there is always a positive probability that the channel experiences a “deep fade” such that it cannot support this rate. In the absence of a non-zero Shannon capacity, one often considers the outage capacity (see, e.g., [8]). When one considers broadcasting a common message to multiple receivers, any event where not all users experience a channel allowing the successful decoding constitutes an outage. In this paper we refer to this specific application where all receivers want the same message as multicasting [1]. Clearly the outage definition is more severe in multicasting. Hence, the outage capacity in such networks is smaller than that in the corresponding unicast scenario. In fact, it is clear that for any target outage probability, as the number of receivers grows, the outage capacity will go to zero. Thus the performance of such a system is severely limited in absence of cooperative diversity.

We show that under the same setup, there is a positive Shannon capacity if we allow cooperative diversity, as the number of users goes to infinity. As noted, when the Shannon capacity is zero, meaningful analysis of system performance can be done using the diversity-multiplexing framework. On the other hand, as in the problem we study, the Shannon capacity is positive, it is of interest to develop protocols that are *capacity* approaching. We refer to such a system with cooperative diversity as “cooperative multicasting”.

Cooperative multicasting has been recently considered by other authors. In [9], the authors have suggested the use of opportunistic large arrays for flooding the network. The basic idea is that each receiver node makes a decision based on the received signal strength. If its decoding threshold exceeds a certain value it becomes active and starts transmitting, otherwise it keeps listening to the signal from other nodes. In [10], the authors extend their work to show that under a fixed relay power per unit area, as the number of users approaches infinity, the network is fully connected if the decoding threshold is set below a critical value and not connected if the threshold is above this value. However, their work focusses on the performance of a specific scheme and does not address the fundamental limits on cooperative multicasting. Furthermore their model considers path loss based on network geometry and does not consider Rayleigh fading.

Our main result is that under a total power constraint P, the cooperative multicasting systems in a network with independent slow Rayleigh fading between any two nodes, have a positive capacity as the number of receivers approaches infinity. More precisely, there is a rate $C = \log\left(1+\frac{P}{N_0}\right) > 0$, such that for all rates $R < C$, the probability of outage (of the worst user) approaches zero as the number of users in the network approaches infinity. Beyond this rate, the probability of outage is bounded away from zero even as the number of receivers approaches infinity. Moreover a relatively simple protocol based on cooperative diversity approaches this bound.

The rest of the paper is organized as follows. In Section 2, we develop a channel model for the cooperative multicasting scenario. Section 3 presents a simple upper bound for the achievable rate of transmission. In Section 4 we describe a transmission protocol that approaches the upper bound. In Section 5, we discuss implications of our results in light of prior work and in Section 6 we present some open problems.

2 Channel Model

We consider a scenario in which one source node wants to broadcast a common message to K receiver nodes. The receiver nodes are numbered $\{1, 2, \dots, K\}$ and the source node

[1]Sometimes the term multicasting is used when only a subset of receivers want the same message. But in this paper, we only consider the case when all the receivers want the same message

is numbered 0. Each receiver node has the ability to transmit a signal under a half duplex constraint. A node cannot transmit and receive in the same time-slot. The channel gain between node $i \in \{0,1,\ldots,K\}$ and node $j \in \{1,2,\ldots,K\}$ is denoted by h_{ij} which is assumed to be $\mathcal{CN}(0,1)$, independent of all other gains and constant throughout. We assume that only the receiver node j knows the channel gain h_{ij} and it is learned when node i begins transmission. Let T_i denote the set of transmitting nodes and L_i denote the set of listening nodes in time-slot i. Then the received signal by node $l \in L_i$ is given by,

$$y_l(i) = \sum_{j \in T_i} h_{jl} x_j(i) + w_l(i) \tag{1}$$

Here w_l is the additive Gaussian noise which is modeled as $\mathcal{CN}(0,N_0)$. The transmitted symbols x_j should be chosen to satisfy the sum power constraint. If the set T_i does not change during the course of the transmission (i.e. T_i is independent of i), then clearly, the sum power constraint is given by $E[\sum_{j\in T_i} |X_j|^2] \leq P$. If the set T_i is not constant throughout the transmission of the message one has to ensure that the average power over all such sets satisfies the power constraint. While designing the cooperative multicasting protocols, the set of transmitting users could change on the fly. One has to explicitly ensure that the sum power constraint is satisfied when the set of transmitting users changes on the fly. In general such a requirement is non-trivial, but in the limit of large number of receivers we will see that the *size* of the transmitting set becomes deterministic and this makes the power normalization relatively straightforward.

Definition 1 *A rate R is achievable for the cooperative multicasting system if for any $\varepsilon > 0$, there is a number K such that a for a system with at least K users, the message W distributed uniformly over $\{1,2,\ldots,2^{nR}\}$ can be decoded by all the receivers, with probability at least $1-\varepsilon$, as the number of channel uses $n \to \infty$.*

Note that in Definition 1, for any K, we are taking the block length n to infinity. Accordingly, we can drive the detection error to a negligible fraction of the outage error probability. The dominant cause of error in ε, above is the probability of error due to outage. To claim that the rate R is achievable, we will have to show that the outage probability approaches zero, as $K \to \infty$.

3 Simple Upper Bound

In this section, we present a simple upper bound on the common rate R in Definition 1. Suppose a genie conveys the message W to receiver nodes $1,2,\ldots K-1$ and only receiver node K remains to be served. This is clearly a MISO system with K transmit antennas, with channel knowledge only at the receiver. Suppose we decide to use i.i.d. input distribution across the antennas (i.e. the input covariance matrix is a scaled identity). As $K \to \infty$, the effective channel gain at the receiver approaches unity $\left(\frac{1}{K}\sum_{i=0}^{K-1} |h_{iK}|^2 \xrightarrow{K\to\infty} 1\right)$. Accordingly, $C = \log\left(1+\frac{P}{N_0}\right)$ is achievable with the outage probability approaching zero as $K \to \infty$. In order to establish that the outage probability is bounded away from zero for all rates $R > C$, we need to invoke the following result for the MISO channel with i.i.d. Rayleigh fading [11].

Fact 1 (MISO Channel) *Consider a MISO channel with K transmit antennas and one receive antenna and i.i.d. Rayleigh fading. The optimal coding strategy that minimizes the outage probability for any fixed rate R is to choose input at each antenna to be independent i.e. $\Lambda_x = \mathrm{diag}\{P_1,P_2,\ldots,P_K\}$ and to allocate powers equally among a subset of antennas i.e. $P_1^* = P_2^* = \cdots = P_{K_0}^* = P/K_0$ and to allocate nothing to the remaining antennas.*

Using the above result, we now show that the outage probability is bounded away from zero for every $R > C$

Proposition 1 *Suppose for some* $\varepsilon > 0$, $R = \log\left(1 + \frac{P(1+\varepsilon)}{N_0}\right)$. *Then necessarily, the outage probability is bounded away from zero according to* $\Pr(\text{outage}) \geq \frac{\varepsilon}{1+\varepsilon}$.

Proof:
Let K_0 be the optimum number of transmitting antennas as described in Fact 1. Also let $G_{K_0} = \frac{1}{K_0}\sum_{i=0}^{K_0-1} |h_{iK}|^2$. The corresponding rate is $R = \log\left(1 + G_{K_0}\frac{P}{N_0}\right)$. Note that G_{K_0} is a scaled chi-squared random variable with unit mean. We have

$$\begin{aligned}
\Pr(\text{outage}) &= 1 - \Pr(G_{K_0} > 1+\varepsilon) \\
&\geq 1 - \frac{1}{1+\varepsilon} \qquad \text{(Markov Ineqality)} \\
&= \frac{\varepsilon}{1+\varepsilon}
\end{aligned}$$

■

Note that the above result is independent of the number of antennas. Thus for $R > C$, the outage probability is bounded away from zero even as $K \to \infty$, and hence this rate is not achievable according to Definition 1. This leads us to the following upper bound:

Lemma 1 *In the limit* $K \to \infty$, *an outer bound on the achievable rate for cooperative multicasting is given by*

$$C^{\text{upper}} = \log\left(1 + \frac{P}{N_0}\right) \tag{2}$$

4 A Cooperative Diversity Based Scheme

In this section, we present a two step scheme which uses cooperative diversity for multicasting in large networks. The main idea in this scheme is to divide the users into two groups. In the first step, the sender broadcasts the message at a high rate, which only a fraction of users (say α) can decode. These users then form a distributed virtual antenna and use a distributed space time code to serve the remaining $1-\alpha$ fraction of users. See Figure 1. The main steps of this scheme are as follows:

(i) The source transmits the message at a rate $R_\alpha = \log(1 + G_\alpha \frac{P}{N_0})$ where $G_\alpha \triangleq F^{-1}(1-\alpha)$ is selected so that on an average a fraction α of the best users can decode the message. Here $F(\cdot)$ denotes the cumulative distribution function of the channel gain $|h_{0i}|^2$.

(ii) Let N be the number of users that were successful in decoding the message in step(i). We declare an error if $\left|\frac{N}{\alpha K} - 1\right| > \varepsilon$ for some fixed $\varepsilon > 0$. The reason for this will become clear in the following steps.

(iii) Each of the N users that successfully decodes the message in step(i), transmits with a power $\tilde{P} = \frac{P}{\alpha K(1+\varepsilon)}$. Since we declare an error if $N > \alpha K(1+\varepsilon)$, we do not exceed the power limit during a successful transmission. The codebooks of each of the K users is generated independent of all other users according to $\mathcal{CN}(0, \tilde{P})$ and rate $\tilde{R} = \log\left(1 + \frac{P(1-\varepsilon)}{(1+\varepsilon)}(1-\delta)\right)$ for some $\delta > 0$. Each successful user transmits the codeword in its codebook corresponding to the received message.

(iv) Each of the remaining $K - N$ receivers performs an exhaustive search over the codebooks of all other users to find a sequence that is jointly typical with the received

Figure 1: Illustration of the two-stage space time coded cooperative diversity scheme. In the first stage, the source node broadcasts at a high rate and only a small number of receiver nodes are able to decode. These nodes are shaded. In the second stage, these nodes cooperatively broadcast the message to the remaining nodes using a suitable space-time code protocol.

sequence [2]. An error occurs if atleast one of the receivers cannot successfully decode the message. If all the decoders are successful then the overall rate achieved by the system is given by $R_{\text{eff}} = \frac{\tilde{R}}{1+\frac{\tilde{R}}{R_\alpha}}$.

4.1 Probability of Error Analysis

We now analyze the probability of error for the two events we described in step (ii) and (iv). As we stated before in Section 2, we assume that the block lengths are large enough so that the dominating cause of error event is outage rather than detection. Accordingly, the number of of users N that can successfully decode the message at rate R_α is given by $N = \sum_{i=1}^{K} \mathbf{1}[|h_{0i}|^2 > G_\alpha]$ (where $\mathbf{1}$ is the indicator function). Since all the channel gains are independent, it follows that N is a binomial random variable, with mean αK and variance $\alpha(1-\alpha)K$. Accordingly, we can use the Chernoff bound for binomial random variables [12], to bound the probability of error in step (ii) as:

$$\begin{aligned} \Pr\left(\left|\frac{N}{\alpha K} - 1\right| > \varepsilon\right) &= \Pr(N > \alpha K(1+\varepsilon)) + \Pr(N < \alpha K(1-\varepsilon)) \\ &\leq e^{-\alpha K \frac{\varepsilon^2}{2}} + e^{-\alpha K \frac{\varepsilon^2}{4}} \qquad (3) \end{aligned}$$

We now calculate the probability of error in step (iv) given that there was no error in step (ii). Again, we only need to focus on the outage error. Since $N \in (\alpha K(1-\varepsilon), \alpha K(1+\varepsilon))$, and each transmitting user has power $\tilde{P} = \frac{P}{\alpha K(1+\varepsilon)}$, each of the remaining user can decode successfully at a rate of $\tilde{R}_i = \log\left(1 + G_i \frac{P(1-\varepsilon)}{(1+\varepsilon)N_0}\right)$, where $G_i = \frac{1}{N}\sum_j |h_{ji}|^2$ is the effective channel gain of user G_i. If we define $G_{\min} = \min_i G_i$, then an outage occurs, if $G_{\min} < 1-\delta$. We show in the Appendix that the probability of this event is bounded by the following lemma.

Lemma 2 *For any* $0 < \alpha < 1$, *suppose* αK *users serve the remaining* $(1-\alpha)K$ *users using the cooperative scheme described above. Let* $G_{\min}$ *be the effective channel gain of*

[2]We assume that these $K-N$ users do not use the received sequence from the source in this step. As we will see, there is no loss in optimality by ignoring this information, in the scale we are interested in.

the worst of the $(1-\alpha)K$ *users as defined above. Then for any* $0 < \delta < 1$*,*

$$\Pr(G_{\min} < 1-\delta) \leq -(1-\alpha)K \log\left(1-\gamma^{\alpha K}\right) \tag{4}$$

Where $\gamma = ((1-\delta)e^{\delta})$*. The probability of error decays exponentially in* K*.*

Finally, the overall probability of error is given by

$$\begin{aligned}
\Pr(\text{error}) &= \Pr(\text{error in (ii)}) + \Pr(\text{success in (ii)}) \Pr(\text{error in (iv)}|\text{success in (ii)}) \\
&\leq \Pr(\text{error in (ii)}) + \Pr(\text{error in (iv)}|\text{success in (ii)}) \\
&= \Pr\left(\left|\frac{N}{\alpha K} - 1\right| > \varepsilon\right) + \Pr(G_{\min} < 1-\delta) \\
&\leq e^{-\alpha K \frac{\varepsilon^2}{2}} + e^{-\alpha K \frac{\varepsilon^2}{4}} - (1-\alpha)K \log\left(1-\gamma^{\alpha K}\right) \qquad \text{(from (3),(4))}
\end{aligned}$$

Thus, for any choice of $\varepsilon > 0$, $\delta > 0$ and for any $0 < \alpha < 1$ such that $\alpha K \to \infty$ as $K \to \infty$, the probability of error decreases exponentially in K. By choosing, ε and δ arbitrarily close to 0, we can make the rate $\tilde{R}$ close to C^{upper} in (2). For the overall rate R_{eff} to approach C^{upper}, we also require that R_α be arbitrarily large. One possible choice of α that satisfies $\alpha K \to \infty$ and $R_\alpha \to \infty$ is $\alpha \sim \frac{1}{O(K)}$. With this choice of α, the effective rate of the system $R_{\text{eff}} = \frac{\tilde{R}}{1+\frac{\tilde{R}}{R_\alpha}}$ can be made arbitrarily close of C^{upper} and the probability of outage decreases exponentially in αK. This result can be stated in the following theorem:

Theorem 1 *In the limit* $K \to \infty$*, the capacity of cooperative multicasting with a sum power constraint of* P *and noise* N_0 *is given by* $\lim_{K\to\infty} C(K) = \log\left(1+\frac{P}{N_0}\right)$.

5 Discussion and Comparison with Prior Work

In the following sections we describe some issues regarding our model and the relation with previous work.

5.1 Size of Group 1

The inner bound we described essentially divides the users into two groups. Users in group 1 get served directly from the source and group 2 users get served by the users in group 1. The outage probability in Section 4 decreases exponentially in the number of users in group 1. So from this point of view, it is better to have more users in group 1. On the other hand, if we have more users in group 1, a larger fraction of time is devoted in serving group 1 (R_α is smaller) and hence we are further away from the capacity. The result of Theorem 1 says that in the limit of $K \to \infty$, we can make the overhead arbitrarily small and still have exponential decay in outage probability and approach the capacity. Note however that this conclusion relies on the fact that the "goodness" of a channel is unbounded. Our analysis assumes that there is a small fraction of users with very strong channel. This is questionable in a realistic channel model. Nonetheless even if we impose some upper bound on how large the channel gains may be, the basic result that the outage event goes to zero as the number of users grows will still hold.

5.2 Channel Modeling Assumptions

Most prior work on cooperative diversity has assumed an i.i.d. Rayleigh fading model between any two pair of nodes [3]-[7]. The implicit assumption is that the receivers are spread out on a large area and do not have line of sight. On the other hand, the work on scaling laws in dense wireless networks [13, 14] usually assumes a network geometry and focuses on path loss instead of fading. Many nodes are assumed to be densely packed in a relatively small area and so line of sight is dominant. In the present work we are considering the limit of large number of users and still considering i.i.d. Rayleigh fading model. One must consider the validity of our assumption in this operating regime. One possible justification is that the outage probability decreases exponentially in the number of users. Hence, even though the result of Theorem 1 holds in the limit of $K \to \infty$, in practice the outage probability will be negligible even with a relatively small number of users. Secondly we note that the main point of this work is to show that in contrast to a non cooperative system, cooperative diversity gains can be used to define the notion of a non-zero capacity in multicasting. A Rayleigh fading model is sufficient to draw this point. A more sophisticated model, such as a hybrid of path loss and fading model can only improve the achievable rates through cooperative diversity in multicasting.

5.3 High SNR vs Large Number of Relays

A number of authors have considered the two step protocol similar to ours but in different settings. Laneman et al. [3] consider a similar scheme in a setting consisting of a single source, a single destination and several relays. The following are the main points of difference with the current scheme.

1. Each relay has an individual power constraint, rather than a total power constraint. The number of relays is finite.
2. The first step, where the relays try to decode the message and the second step where they cooperate to serve the destination take equal time. This is equivalent to setting $\alpha = 0.5$
3. The performance is based on diversity-multiplexing tradeoff and is valid in the high SNR regime.

The setup of [3], it is shown that the diversity gain is of the order of the number of relays (not the number of relays that were successful in decoding the message in step 1). Here, diversity is explicitly quantified as it indicates the robustness one achieves for a *fixed* multiplexing gain in the high SNR regime. In the present work, we focus on the limit of large number of users and fixed average sum power. In this regime we show that cooperative diversity works to our advantage in defining the notion of capacity. We observe that in this regime, there is enough diversity to serve *every* user in the system with probability of outage approaching zero as long as the rate is below this capacity and conversely we cannot have an arbitrarily low outage if we operate above the capacity.

5.4 Fixed sum power vs individual power constraint

We have imposed a sum power constraint across the network. Since the set of transmitting relays is not decided apriori, it is could be challenging to design cooperative protocols with a sum power constraint. In this work, we exploited the fact that as the number of users becomes large, the size of the users in group 1 becomes deterministic and the sum power constraint can be satisfied through a fixed power allocation. The main point of imposing a sum power constraint across the users is to separate the diversity gains from the gains by increasing power with the number of users. On the other hand, if an individual power constraint is imposed, then the total power increases linearly with the number of users. This will result in even higher achievable rates.

6 Conclusion and Future Work

While most of the previous work on cooperative diversity has focussed on the diversity-multiplexing tradeoff at high SNR, our approach considers a different operating regime: We fix the total power across the network and let the number of users go to infinity. In this regime, we show that there is a notion of capacity in a sense that even in slow fading environments the outage probability goes to zero as the number of users goes to infinity for all rates below capacity. We derived the capacity for cooperative multicasting with independent Rayleigh fading channels and proposed a simple scheme that approaches this capacity.

Future Work

One direction for future work is to consider more sophisticated channel models that improve upon the i.i.d. Rayleigh fading model considered in this work. In particular, if the number of users becomes large, one must also take the network geometry into account. As discussed in Section 5, such a model will require a careful combination of the Rayleigh fading and path loss models which have mostly been considered independently in prior work. In particular the assumption of independent path gains between users may not hold. It will be instructive to consider capacity theorems for such channel models. While such models will improve upon the achievable rates due to cooperative diversity, the inner bound presented in Section 4 may not be optimal under these conditions.

It may be useful to consider not only the asymptotic performance of different schemes in the limit $K \to \infty$, but also study the decay of outage probability. In particular, the quantity $C(R) = -\frac{\log \Pr\{\text{outage}\}}{K}$ defines the tradeoff between the outage decay and the common message rate in the limit of large number of users. Such a tradeoff provides a framework for analyzing different schemes that are asymptotically optimal. Having our proposed inner bound as a baseline scheme it would be useful to consider the performance of other schemes in the framework of this tradeoff.

Another direction under investigation is the use of a feedback protocol in the system. The main advantage of a feedback protocol is that we can drive the outage probability to zero for a fixed number of users. It might be worth investigating the fundamental limits of such “zero outage” capacity and propose practical feedback based schemes that approach this capacity.

Finally we note that the present work only considers an information theoretic treatment of cooperative diversity. To realize the gains promised by such protocols, one has to design practical space time codes that approach these limits. Other issues such as maintaining perfect synchronization between the receivers and having perfect channel knowledge at the receiver are also particularly challenging from a practical viewpoint.

Appendix: Proof of Lemma 2

In this section, we prove the result of Lemma 2. We prove a slightly different result and modify it to get the result of the lemma. Suppose $0 < \alpha < 1$, such that $\alpha K \to \infty$ as $K \to \infty$. Then for any $0 < \delta < 1$

$$\Pr\left\{ \min_{i \in \{1,2,\ldots,\bar{\alpha}K\}} G_i > 1 - \delta \right\} > 1 + \bar{\alpha} K \log(1 - \gamma^{\alpha K})$$

where $\gamma = (1-\delta)e^{\delta}$ and $\bar{\alpha} = 1-\alpha$.

$$\begin{aligned}
& \Pr\left\{\min_{i\in\{1,2,\ldots,\bar{\alpha}K\}} G_i > 1-\delta\right\} \\
= & \left(\Pr\{G_i > 1-\delta\}\right)^{\bar{\alpha}K} && \text{(since all the channel gains are i.i.d.)} \\
= & \left(1-\Pr\{G_i \le 1-\delta\}\right)^{\bar{\alpha}K} \\
= & \left(1-\Pr\{e^{-sG_i} \ge e^{-s(1-\delta)}\}\right)^{\bar{\alpha}K} && \text{(for any s > 0)} \\
\ge & \left(1-E[e^{-sG_i}]e^{s(1-\delta)}\right)^{\bar{\alpha}K} && \text{(Markov Inequality)} \\
= & \left(1-(E[e^{-s\hat{g_{ij}}}])^{\alpha K}e^{s(1-\delta)}\right)^{\bar{\alpha}K} && \text{(Where } \hat{g_{ij}} = \frac{1}{\alpha K}g_{ij}) \\
= & \left(1-\frac{e^{s(1-\delta)}}{(1+\frac{s}{\alpha K})^{\alpha K}}\right)^{\bar{\alpha}K} && \text{(If } X \text{ is Exponential(1), } E[e^{-sX}] = \frac{1}{1+s})
\end{aligned}$$

The above relation holds for any $s > 0$. We select $s = \alpha K\left(\frac{1}{1-\delta}-1\right)$. Accordingly, $s(1-\delta) = \alpha K\delta$ and $1+\frac{s}{\alpha K} = \frac{1}{1-\delta}$. Hence we have that

$$\Pr\left\{\min_{i\in\{1,2,\ldots,\alpha K\}} G_i > 1-\delta\right\} \ge \left(1-((1-\delta)e^{\delta})^{\alpha K}\right)^{\bar{\alpha}K}$$

Define $\gamma \stackrel{\Delta}{=} (1-\delta)e^{\delta}$. For $\delta \in (0,1)$, we have $\gamma \in (0,1)$.

$$\begin{aligned}
& \Pr\left\{\min_{i\in\{1,2,\ldots,\alpha K\}} G_i > 1-\delta\right\} \\
\ge & \left(1-\gamma^{\alpha K}\right)^{\bar{\alpha}K} \\
= & e^{\bar{\alpha}K\log(1-\gamma^{\alpha K})} \\
\ge & 1+\bar{\alpha}K\log(1-\gamma^{\alpha K}) && \text{(Since } e^{-x} > 1-x \text{ for } x \in (0,1))
\end{aligned}$$

It follows that,

$$\Pr\left\{\min_{i\in\{1,2,\ldots,\alpha K\}} G_i \le 1-\delta\right\} \le -\bar{\alpha}K\log(1-\gamma^{\alpha K})$$

Finally, note that since $\log(1-\gamma^{\alpha K}) \approx -\gamma^{\alpha K}$ for large αK, we have that

$$\lim_{K\to\infty} -\bar{\alpha}K\log(1-\gamma^{\alpha K}) = 0,$$

$$\lim_{K\to\infty} \frac{\log(-\bar{\alpha}K\log(1-\gamma^{\alpha K}))}{K} = \alpha\log\gamma < 0$$

The probability of outage decays exponentially in K.

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