Structured Space-Time Block Codes With Optimal Diversity-Multiplexing Tradeoff and Minimum Delay

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***Abstract*— It is well-known that using multiple antennas can substantially increase the data rate and robustness of communication systems in a fading environment. It was recently established that there is a tradeoff between these two types of gains, termed diversity-multiplexing tradeoff. In this paper, we develop a family of short structured space-time block codes that achieves the optimal tradeoff for the two-transmit two-receive antenna systems with the minimum delay of two necessary for optimality. It uses the idea of rotation of cross-diagonal entries of an uncoded system to achieve spreading of information across space and time to obtain the maximum diversity while preserving multiplexing gain. Rotation angles that are optimal in terms of a determinant criterion and universal for all rates are identified. Performance analysis and simulation results are presented to demonstrate the achieved tradeoff.**

I. Introduction

For the past few years, people have known that using multiple antennas can substantially increase the capacity and robustness of a communications system in a fading environment. For example, compared to single antenna systems, using two antennas at both ends allows capacity to grow twice as fast with SNR or allows error probability to decay four times as fast. Much work has been done toward achieving each type of gain.

However, Zheng and Tse [1] recently established that there is a tradeoff between these two types of gains, which they named *diversity-multiplexing tradeoff*. (We will refer to it as d-r tradeoff for reasons shown later.) They analytically evaluate the optimal tradeoff curve and show that it is achievable using Gaussian random codes.

They also analyze the d-r tradeoff achieved by some existing deterministic coding schemes. They show that OSTBC (orthogonal space-time block code) achieves the maximum diversity but not multiplexing gain; V-BLAST (vertical Bell labs layered space-time architecture) achieves the maximum multiplexing but not diversity gain; and D-BLAST (diagonal BLAST), which is not a block code, when used with MMSE decoder, achieve the optimal tradeoff curve if we ignore the overhead.

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There have been a variety of efforts to simultaneously achieve good error probability and rate. One approach uses linear combinations of basis matrices to build codebooks, such as [2], [3]. However, the code parameters are often individually optimized for different rates, making code design difficult as rate becomes large.

Another approach uses rotation of QAM constellations to obtain diversity while preserving multiplexing gain. Our design, which we refer to as *tilted-QAM codes*, falls in this category. In the past, this idea was also used in [4] for single antenna communication over multiple fades and later extended to the multiple-antenna single-fade case in [5], [6], in which the end points of the optimal d-r tradeoff are shown to be achieved.

In this paper, we develop a short space-time block code construction for two-transmit two-receive antenna systems with code duration two that achieves the entire optimal d-r tradeoff. Rotation matrices are chosen using the determinant criterion in [7], i.e., the determinants of difference matrices are to be kept as far away from zero as possible. With this criterion, the family of codes constructed has a set of universally optimal rotation angles and has the same worst case determinant for all rates. We show that the minimum distances between codewords resulted from having non-vanishing worst case determinant can allow our design to achieve the entire d-r tradeoff frontier. We note that this is also done with the minimum delay of two necessary, which is not achievable using Gaussian random codes. At the end, we compare the performance of our tilted-QAM codes with OSTBC.

We model the multiple antenna channel as flat and Rayleigh fading, having size $N_t \times N_r$, $N_t \leq N_r$, and is known at the receiver but not at the transmitter. We also model the channel as block fading with coherence time longer than the code duration T so that each codeword experiences only one channel realization. When the transmission rate is R b/s/Hz, there are 2^{RT} codeword matrices to be designed. More specifically, we use the system model $\mathbf{Y} = \mathbf{HX} + \mathbf{W}$, where $\mathbf{H}$ is the $N_r \times N_t$ channel matrix with IID entries with zero mean, unit variance, complex Gaussian density, $\mathcal{CN}(0, 1)$, $\mathbf{X}$ is the $N_t \times T$ transmitted signal matrix, entries of $\mathbf{W}$ are IID with density $\mathcal{CN}(0, 2\sigma_w^2)$, and $\mathbf{Y}$ is the received signal matrix.

II. Diversity-Multiplexing Tradeoff

Let us first recall the definition of diversity and multiplexing gains introduced by Zheng and Tse [1] and the associated optimal tradeoff of interest.

For a given SNR and transmission rate $R(\mathrm{SNR})$, let $P_e(R(\mathrm{SNR}), \mathrm{SNR})$ be the error probability achieved. Diversity and multiplexing gains are defined to be how fast error probability decays and rate increases with SNR,

$$d = -\lim_{\mathrm{SNR}\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} \tag{1}$$

and

$$r = \lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log_2 \mathrm{SNR}}. \tag{2}$$

Their main result states that there is an intrinsic tradeoff between these two quantities. The optimal tradeoff, $d_o(r)$, achievable by any $N_t \times N_r$ system, is given by the piece-wise linear function connecting the points $(k, d_o(k))$, $k = 0, 1, \cdots, \min(N_t, N_r)$, where

$$d_o(k) = (N_t - k)(N_r - k). \tag{3}$$

For the case where $N_t = N_r = 2$, the tradeoff curve is plotted in Fig. 1 as a solid line. Note, the maximum diversity is four and the maximum multiplexing is two. In the rest of this paper, we focus only on this 2×2 case.

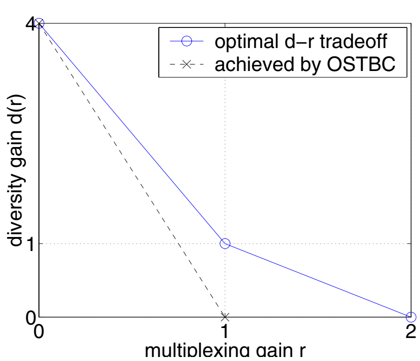

Fig. 1. Diversity-multiplexing tradeoff curves in $N_t = N_r = 2$ case.

A method we use to interpret and visualize the tradeoff is by looking at a family of error probability curves for different rates. If we allow powerful and infinitely long codes, then the error probability reaches the achievable lower bound, which is the channel outage probability, $P_{\mathrm{out}}(R, \mathrm{SNR}) = \Pr[C(\mathbf{H}, \mathrm{SNR}) < R]$, where [1]

$$C(\mathbf{H}, \mathrm{SNR}) = \log_2(\det(\mathbf{I} + (\mathrm{SNR}/N_t)\mathbf{H}\mathbf{H}')) \tag{4}$$

is the channel mutual information, the maximum achievable rate, for IID Gaussian input distribution.

We evaluate $P_{\mathrm{out}}(R, \mathrm{SNR})$ for R=$1, 2, \cdots, 30$ b/s/Hz and 0 dB $\leq$ SNR $\leq$ 60 dB and plot the result in Fig. 2 as a set of dense gray lines, one for each rate. The limiting slope of each curve is four, which is the maximum diversity gain achievable for fixed rates, i.e., $r = 0$. For a fixed P_{out}, at large SNR, the horizontal separation is 2 b/s/Hz per 3 dB, which is the maximum multiplexing gain.

To visualize other points on the d-r tradeoff curve, we choose several values of r. As SNR increases, for rates $R = r \log_2(\mathrm{SNR})$, we determine the corresponding $P_{\mathrm{out}}(R, \mathrm{SNR})$ and obtained another set of outage probability curves (dark) cutting across the first set (gray). We can see that when rate increases more slowly with SNR, the outage probability decays faster. The limiting slope of each curves corresponds to the diversity gain for each r, $d_o(r)$, which are indicated using the dashed lines. We can see that theory and simulation are in agreement.

Note that when rate increases with SNR, the codebook used must also change instead of remain constant. So each (dark) curve is really achieved by a family of codes.

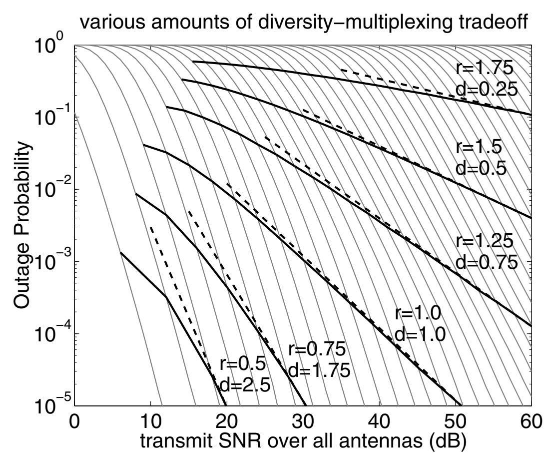

Fig. 2. $P_{\mathrm{out}}(R(\mathrm{SNR}), \mathrm{SNR})$ for fixed R and $R = r \log_2(\mathrm{SNR})$.

Although infinitely long codes are needed to reach the channel outage probability limit (4), only finite length codes are required to achieve the optimal d-r tradeoff. When $N_t = N_r = 2$, the code duration must be at least two. Zheng and Tse [1] show that Gaussian random codes with $T = 3$ can achieve the optimal tradeoff, but not with $T = 2$. We show that it is indeed possible to achieve the optimal tradeoff in the $T = 2$ case with a structured code.

III. OSTBC

A well-known space-time block code is the *orthogonal space-time block code* (OSTBC) [8], [9]. OSTBC can be considered as a smart and short, $T = N_t$, repetition code. For $N_t = N_r = T = 2$, two informations symbols chosen out of regular constellations, such as QAM, are encoded in the following fashion,

$$\mathbf{X} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}, \tag{5}$$

where $(\cdot)^*$ indicates conjugation.

By using repetition to spread each symbol across space and time, OSTBC can achieve the maximum diversity. To see this, we need to verify that all difference matrices are full rank [7]. Without loss of generality, let us assume $\mathbf{0}$ is one of the code matrices, so we can look at the non-zero codeword matrices instead. We have

$$\det(\mathbf{X}) = |s_1|^2 + |s_2|^2 \neq 0 \text{ when } \mathbf{X} \neq \mathbf{0}. \quad (6)$$

Therefore, OSTBC achieves the maximum diversity of $N_t N_r = 4$ when $r = 0$, i.e., error probability decays like SNR^{-4} when rate is kept constant.

However, due to the repetition, only one new symbol is transmitted at a time, so only $r = 1$ can be achieved when $d = 0$, i.e., for a fixed target error probability, R increases by one for every 3 dB increase in SNR. Zheng and Tse [1] show that the d-r tradeoff achievable by an OSTBC system is a straight line between $(r, d) = (0, 4)$ and $(1, 0)$ as shown in Fig. 1 with a dashed line, which is below the optimal tradeoff curve.

In summary, OSTBC achieves the maximum diversity gain but sacrifices multiplexing gain. Our goal is to improve upon OSTBC and achieve the optimal d-r tradeoff.

IV. Tilted-QAM Design

We study a class of tilted-QAM designs, which replaces the repetition in OSTBC with a rotation. For a given transmission rate $R = r \log_2(\mathrm{SNR})$, we use a M^2-QAM constellation carved from $\mathbb{Z} + \mathbb{Z}j$ with size $M^2 = 2^{R/2} = \mathrm{SNR}^{r/2}$. Then, four, instead of two, information symbols, s_{ij}, are encoded into a the codeword matrix $\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$ via two rotations,

$$\begin{aligned} \begin{bmatrix} x_{11} \\ x_{22} \end{bmatrix} &= \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \end{bmatrix}, \\ \begin{bmatrix} x_{21} \\ x_{12} \end{bmatrix} &= \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) \end{bmatrix} \begin{bmatrix} s_{21} \\ s_{12} \end{bmatrix}. \end{aligned} \quad (7)$$

While the rotation avoids the multiplexing gain penalty, we must make all non-zero codeword matrices (equivalent to all difference matrices) full rank to ensure maximum diversity (when $r = 0$). Let the worst case determinant be $\gamma \stackrel{\text{def}}{=} \min_{\mathbf{X} \neq \mathbf{0}} |\det(\mathbf{X})|$. We need to choose the two rotation angles to maximize γ.

Let us first look at $\det(\mathbf{X})$ as a function of (θ_1, θ_2).

$$\begin{aligned} 2\det(\mathbf{X}) \; &= \; \sin(2\theta_1)(s_{11}^2 - s_{22}^2) + 2\cos(2\theta_1) s_{11} s_{22} \quad (8) \\ & \quad - \sin(2\theta_2)(s_{21}^2 - s_{12}^2) - 2\cos(2\theta_2) s_{12} s_{21}. \end{aligned}$$

In order to study the d-r tradeoff in the high SNR limit, we need to know explicitly how the optimal angles depend on rate at arbitrarily high rates. This precludes a brute force search for the optimal angles for each rate.

Interestingly, we find that there is one pair of rotation angles that maximizes γ for all constellation sizes. Thus, we have a universally optimal design for all rates. This result is summarized in the following theorem.

Theorem 1: For codeword matrix $\mathbf{X}$ defined in (7), and any QAM constellation carved from $\mathbb{Z} + \mathbb{Z}j$, the maximum worst case determinant of difference matrices is

$$\max_{(\theta_1, \theta_2)} \min_{\mathbf{X}_1 \neq \mathbf{X}_2} |\det(\mathbf{X}_1 - \mathbf{X}_2)| = \frac{1}{2\sqrt{5}}, \quad (9)$$

and achieved by $(\hat{\theta}_1, \hat{\theta}_2) = \left(\frac{1}{2}\arctan(\frac{1}{2}), \frac{1}{2}\arctan(2)\right)$.

Proof: For binary constellation, by listing all $\det(\mathbf{X})$ expressions for all s_{ij} 4-tuples, we can easily show that $(\hat{\theta}_1, \hat{\theta}_2)$ and its symmetric variations are optimal, with which $\gamma = 1/(2\sqrt{5})$ and is obtained at, for example, $(s_{11}, s_{12}, s_{21}, s_{22}) = (1, 0, 0, 0)$. As constellation grows (without re-normalization), γ could only decrease or remain constant, since there are additional codewords to minimize over. So to prove Theorem 1, it suffices to show that $\gamma = 1/(2\sqrt{5})$ is actually achievable for larger constellations using $(\hat{\theta}_1, \hat{\theta}_2)$, i.e., $|\det(\mathbf{X})| \geq 1/(2\sqrt{5})$ for all non-zero 4-tuples of s_{ij}.

Substituting $(\hat{\theta}_1, \hat{\theta}_2)$ into (8), we have,

$$\begin{aligned} J \; &\stackrel{\text{def}}{=} \; 2\sqrt{5}\det(\mathbf{X}) \quad (10) \\ &= \; s_{11}^2 - s_{22}^2 + 4 s_{11} s_{22} + 2 s_{12}^2 - 2 s_{21}^2 - 2 s_{21} s_{12}. \end{aligned}$$

Since $s_{ij} \in \mathbb{Z} + \mathbb{Z}j$, so is J. Now we need to prove $J \neq 0$, i.e., $|J| \geq 1$, unless all $s_{ij} = 0$. Let us perform completion of squares and change of variables. Let $a \stackrel{\text{def}}{=} s_{11} + 2s_{22}$, $b \stackrel{\text{def}}{=} s_{22}$, $c \stackrel{\text{def}}{=} 2s_{12} - s_{21}$, and $d \stackrel{\text{def}}{=} s_{21}$, then $2J = 2a^2 - 10b^2 + c^2 - 5d^2$. Now we need to prove $2a^2 + c^2 = 5(2b^2 + d^2)$ only when $a = b = c = d = 0$, which requires the following lemma.

Lemma 1: For $x, y \in \mathbb{Z} + \mathbb{Z}j$, if $5 | 2x^2 + y^2$, then $5|x$, $5|y$, and $25 | 2x^2 + y^2$. [1]

Proof: Let $x = 5q_x + r_x$ and $y = 5q_y + r_y$, such that, $q_x, q_y \in \mathbb{Z} + \mathbb{Z}j$ and $r_x, r_y \in \{0, 1, 2, 3, 4\} + \{0, 1, 2, 3, 4\}j$. $5 | 2x^2 + y^2$ implies $5 | 2r_x^2 + r_y^2$. It is straight forward to verify that the only case where $5 | 2r_x^2 + r_y^2$ is $r_x = r_y = 0$. Therefore, $5|x, 5|y$, and $25 | 2x^2 + y^2$. ■

Now using Lemma 1, we can show that

$$\begin{aligned} & 2a^2 + c^2 = 5\left(2b^2 + d^2\right) \quad (11) \\ \Rightarrow \; & 5 | 2a^2 + c^2 \Rightarrow 5|a, \; 5|c, \; 25 | 2a^2 + c^2 \\ \Rightarrow \; & 5 | 2b^2 + d^2 \Rightarrow 5|b, \; 5|d, \; 25 | 2b^2 + d^2 \end{aligned}$$

Therefore, all a, b, c, and d are divisible by 5. Thus, we can divide both sides of (11) by 5^2 and obtain an essentially identical equation, $2a'^2 + c'^2 = 5\left(2b'^2 + d'^2\right)$, where $a', b', c', d' \in \mathbb{Z} + \mathbb{Z}j$. We can repeat the above argument and divide both sides by 5^2 indefinitely. Thus, the only possible solution is $a = b = c = d = 0$. ■

[1] For complex integers, divisibility by a real integer (denoted by $|$) is defined as both real and imaginary parts being divisible.

V. PERFORMANCE/DISTANCE PROPERTY ANALYSIS

In this section, we evaluate the performance of the tilted-QAM design by studying the minimum distance between received constellation points given a particular channel realization. Assume $\mathbf{0}$ is transmitted, for a given $\mathbf{H}$, the distance between codewords $\mathbf{X}$ and $\mathbf{0}$ is $\|\mathbf{HX}\|$, where $\|\cdot\|$ is defined as $\|\mathbf{X}\|^2 = \sum_{i,j} |x_{ij}|^2$. If we can guarantee that $\|\mathbf{HX}\|$ is at least a certain value, $\delta(\mathbf{H})$, for all $\mathbf{X} \neq \mathbf{0}$, then all the received constellation points must be at least distance δ apart. This in turn implies that if the magnitude of the noise is less than $\delta/2$, then a minimum distance decoder can guarantee to decode correctly.

We first identify $\delta(\mathbf{H})$ as a function of $|\det(\mathbf{H})|$ and $\|\mathbf{H}\|^2$. We then relate two expressions, how large $\delta(\mathbf{H})$ is compared to noise level, $\delta^2(\mathbf{H})/\sigma_w^2$, and how large the realized channel mutual information is compared to rate, $2^{C(\mathbf{H})-R}$. We will see that when the channel is not in outage, our system tends to have large distances between codewords, which implies good performance. This suggests that the tilted-QAM code can effectively achieve the optimal d-r tradeoff exhibited by the outage probability curves in Fig. 2. See [11] for a more complete proof.

To lower bound $\|\mathbf{HX}\|^2$ using $|\det(\mathbf{H})|$, we use the minimum determinant property built into the design.

$$\|\mathbf{HX}\|^2 \geq 2|\det(\mathbf{HX})| \geq \frac{1}{\sqrt{5}}|\det(\mathbf{H})|. \tag{12}$$

To lower bound $\|\mathbf{HX}\|^2$ using $\|\mathbf{H}\|^2$, let $\lambda_1 \geq \lambda_2$ be the two singular values of $\mathbf{X}$. Then,

$$\|\mathbf{HX}\|^2 \geq \lambda_2^2\|\mathbf{H}\|^2. \tag{13}$$

Next, we lower bound λ_2^2 using the finiteness of the constellation. Recall that $M^2 = 2^{R/2} = \mathrm{SNR}^{r/2}$, then,

$$\left.\begin{array}{r} |\det(\mathbf{X})| = \lambda_1 \cdot \lambda_2 > \frac{1}{2\sqrt{5}} \\ \lambda_1^2 \leq \|\mathbf{X}\|^2 \leq 8\left(\frac{M}{2}\right)^2 = 2\mathrm{SNR}^{r/2} \end{array}\right\} \Rightarrow \lambda_2^2 \geq \frac{\mathrm{SNR}^{-r/2}}{40}. \tag{14}$$

The noise level can be expressed as

$$\sigma_w^2 = \frac{M^2/12}{\mathrm{SNR}/2} = \frac{2^{R/2}}{6\,\mathrm{SNR}} = \frac{\mathrm{SNR}^{r/2-1}}{6}. \tag{15}$$

Combine (12),(13), (14), and (15), we have

$$\frac{\delta^2(\mathbf{H})}{\sigma_w^2} \geq \max\left(\frac{\mathrm{SNR}^{1-r/2}|\det(\mathbf{H})|}{\sqrt{5}/6}, \frac{\mathrm{SNR}^{1-r}\|\mathbf{H}\|^2}{20/3}\right). \tag{16}$$

The channel mutual information for each $\mathbf{H}$ realized was given in (4). When $N_t = N_r = 2$, from (4), we obtain,

$$2^{C-R} \approx \left(\frac{\mathrm{SNR}^{1-r/2}|\det(\mathbf{H})|}{2}\right)^2 + \left(\frac{\mathrm{SNR}^{1-r}\|\mathbf{H}\|^2}{2}\right). \tag{17}$$

Comparing (16) and (17), the expressions are similar up to some constant factors. This difference can be ignored when we are only concerned with d-r tradeoff, which focus on the high SNR regime. We can see that when $C(\mathbf{H})$ is large compared to R, one of $|\det(\mathbf{H})|$ and $\|\mathbf{H}\|^2$ must be large. Consequently, $\delta^2(\mathbf{H})$ is large compared to σ_w^2. Therefore, we have low probability of error when the channel is not in outage. Thus, the tilted-QAM code can effectively achieve the optimal d-r tradeoff. We note that this result is obtained by exploiting the worst case determinant remaining a non-vanishing distance away from zero as rate increases in (12) and (14).

One might wonder whether the constant factor difference, along with the finite length of the code, would preclude error probabilities approaching outage probabilities. However, numerical simulations presented next show that the error rate gaps are actually quite small.

VI. SIMULATION RESULT

Using the tilted-QAM design (7), four symbols of $s_{ij} \in \mathbb{Z} + \mathbb{Z}j$ are encoded into a 2×2 matrix $\mathbf{X}$, which is then transmitted over the multiple antenna channel, $\mathbf{Y} = \mathbf{HX} + \mathbf{W}$. At the receiver, to decode s_{ij} from $\mathbf{Y}$, we must deal with the combined effect of the encoder and the channel. The overall channel turns out to be linear and is equivalent to a sequence of matrix multiplications involving rotation, permutation, and channel distortion. For this channel, we can efficiently implement maximum likelihood decoding using sphere decoding techniques [10].

We run simulations for rates $R = 2N_t \log_2(M) = 4, 8, 12, \cdots, 32$ b/s/Hz, which require M^2-QAM constellations of sizes $M = 2, 4, 8, \cdots, 256$. Random channels with IID $\mathcal{CN}(0,1)$ entries are generated for each trial. The resulting 2×2 block error rate vs SNR curves are plotted in Fig. 3. The outage probability curves for the same rates are also plotted for comparison.

We see that the simulated block error rate curves have similar slopes and separation as the outage probability curves. This suggests that the tilted-QAM design effectively achieves the optimal d-r tradeoff. For instance, at high SNR and relatively high error rate, the separation between the curves is 6 dB, which corresponds to the maximum multiplexing gain of 2 bits per 3 dB. At low SNR and sufficiently low error rate, the slope of each curve approaches four, which is the maximum diversity gain. As we see, the gap between the two sets of curves is quite small, especially at high rates.

Next, we look at the performance of OSTBC, which encodes two symbols into a codeword matrix of $\mathbf{X}$ according to (5). We run simulations for rates $R = 2N_t \log_2(M) = 4, 8, 12, 16$ b/s/Hz, which require M^2-QAM constellations of sizes $M = 4, 16, 64, 256$. Compared to tilted-QAM, OSTBC needs larger constellations

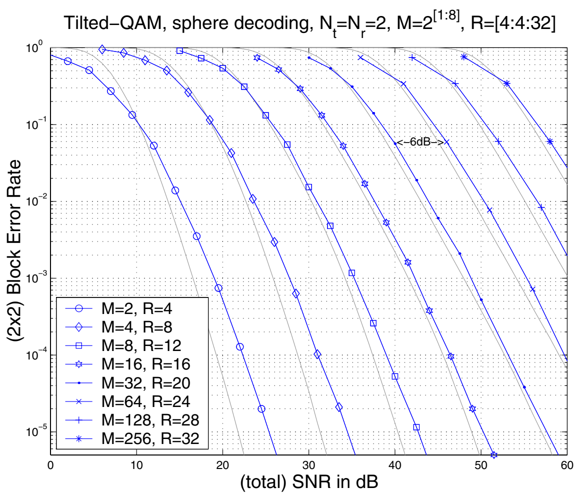

Fig. 3. Error rate curves of titled-QAM code for various rates compared with outage probability curves. The curves have similar slopes and separation, which implies similar diversity-multiplexing tradeoff.

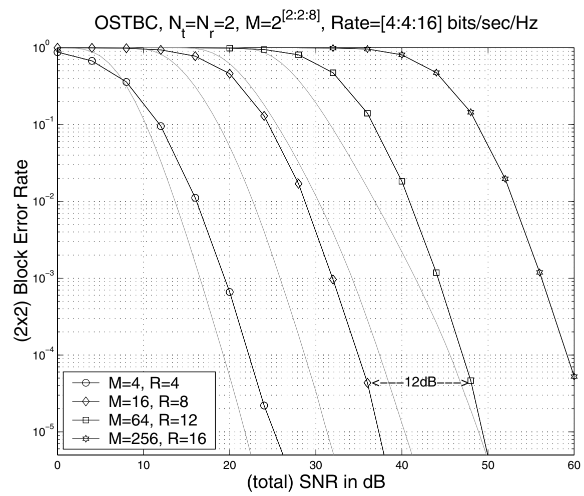

Fig. 4. Error rate curves of OSTBC code for various rates compared with outage probability curves. The separation between the curves is much larger compared to tilted-QAM codes.

for the same rates due to the repetition. This reflects the multiplexing gain loss experienced by OSTBC. The resulting 2×2 block error rate vs SNR curves are plotted in Fig. 4, together with the corresponding outage probability curves.

We see that the slope of each curve approaches four, which is the maximum diversity gain. The separation between the curves is 12 dB or 1 bit per 3 dB, corresponding to a multiplexing gain of $r = 1$ when $d = 0$. Compared to the underlying outage probability curves, OSTBC becomes further from optimal as rate increases, which is due to the loss of multiplexing gain.

Comparing tilted-QAM and OSTBC performance, we see that at 4 b/s/Hz, they are similar. For rates below 4 b/s/Hz, OSTBC is near optimal and is preferred for its lower decoding complexity. As rate increases, tilted-QAM codes out perform OSTBC by increasing amounts due to the superior multiplexing gain. Tilted-QAM codes achieve the same rates at much lower SNR and its structure conveniently lends itself to sphere decoding.

VII. Summary and Remarks

In this paper, we presented a tilted-QAM design that can achieve the optimal diversity-multiplexing tradeoff for two-transmit two-receive antennas systems. It also has a duration of two, thereby, answering the previously open question of whether the optimal tradeoff is achievable at this length. Further detail can be found in [11].

This paper focused only on the two-transmit two-receive antenna case. While it is relatively easy to accommodate more receive antennas, code design becomes much more difficult when there are more antennas at both ends. In [6], full-rank and full-rate codes are designed, but there is no guarantee on the worst case determinant. In [4], determinant issues are studied, but for the single-antenna multiple-fade problem. Even so, some insights there may still be useful. As of now, code design maximizing worst case determinant is still an open problem.

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