Comparing Application- and Physical-Layer Approaches to Diversity on Wireless Channels

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***Abstract*— Diversity techniques often arise as appealing means for improving performance of multimedia communication over certain types of channels with independent parallel components (*e.g.*, multiple antennas, frequency bands, or time slots). Diversity can be obtained by channel coding across parallel components at the physical layer. Alternatively, the physical layer can present an interface to the parallel components as separate, independent links thus allowing the application layer to implement diversity in the form of multiple description source coding. We compare these two approaches in terms of average end-to-end distortion as a function of channel signal-to-noise ratio (SNR). When specialized to the case of an independent, identically distributed Gaussian source over Rayleigh fading channels, our results suggest that parallel channel coding at the physical layer is more efficient than independent channel coding combined with multiple description source coding. More generally, we provide intuitive guidelines for allowing system designers to identify which types of systems are preferable under different scenarios of practical interest.**

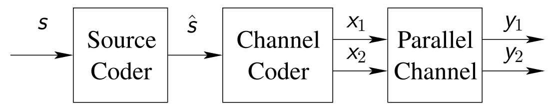

Fig. 1. Diagram for a system with single description source coding combined with parallel channel coding.

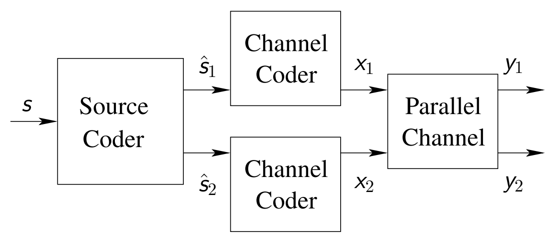

Fig. 2. Diagram for a system with multiple description source coding combined with independent channel coding.

I. Introduction

In wireless links, effects such as fading, shadowing, interference from other transmitters, and congestion can cause the channel quality to fluctuate dramatically potentially introducing distortions into the received multimedia stream. When channel fluctuations are ergodic, it is well-known that limiting performance can be achieved by averaging over channel variations provided suitably long delays are allowed. Since long delays are generally unacceptable in multimedia communication, a wide variety of techniques have emerged to combat channel uncertainty in delay-constrained, or non-ergodic, settings.

The source-channel separation theorem does not apply to non-ergodic channel models. Hence separate design of source and channel coding is generally sub-optimal and achieving the best performance requires a joint source-channel coding approach. While a number of special cases such as progressive and multiple description source codes [1], [2], broadcast channel codes [3], and hybrid analog-digital codes [4, Chapter 3], have been studied, the general problem remains unsolved.

Practical systems have often been designed to combat channel uncertainty by exploiting diversity either at the physical layer via channel coding (*e.g.*, space/time/frequency diversity) [5] or at the application layer via source coding (*e.g.*, multiple descriptions coding) [2], [6]. To investigate the benefits possible with these different approaches, we consider two systems communicating over a parallel channel which could, for example, represent separate frequency bands or time-slots.

A. System Configurations

In the single description system of Fig. 1, a source s is encoded into $\hat{s}$ by the source coder. Next $\hat{s}$ is jointly encoded into (x_1, x_2) by the channel coder and transmitted across a parallel channel. Each channel is represented by a family of probability distributions $p(y_i|x_i; a_i)$ parameterized by a_i; here, a_i might model fading, shadowing, in-

This work has been supported in part by Hewlett-Packard through the MIT/HP Alliance and by NSF under Grant No. CCR-9979363 as well as through NSF Graduate Research Fellowships. * Dr. Laneman is now with the EE Dept. at the University Notre Dame terference, or congestion. The channel decoder attempts to decode $\hat{s}$ from the channel outputs (y_1, y_2).

In the multiple description system of Fig. 2, a source s is encoded into $\hat{s}_1$ and $\hat{s}_2$ by the source coder. Each $\hat{s}_i$ is then separately encoded into x_i and transmitted across the appropriate channel. The channel decoders receive the channel outputs y_i and attempt to decode $\hat{s}_i$. If only one of the $\hat{s}_i$'s can be recovered, the resulting codeword is used to produce a low fidelity source reconstruction $\hat{s}$. If both $\hat{s}_i$'s are successfully decoded they are combined to form a high fidelity reconstruction again denoted by $\hat{s}$.

Single description systems achieve diversity in the sense that if one channel is bad, $\hat{s}$ can still be recovered provided the composite channel is good enough. By contrast, multiple description systems achieve diversity in a different manner. If one channel is bad the source codeword for that channel might be lost, but the source codeword for the other channel can still be recovered and a low fidelity reconstruction of the source can be obtained.

B. Overview of Results

While the delay constraints of the system might allow us to approach performance limits in the source coder, we consider the scenario in which these constraints are stringent enough that the channels exhibit only a single realization of the parameters a_i (*e.g.* the fading coefficients) during the channel coding interval. Thus, we cannot guarantee *a priori* that any fixed transmission rate $R > 0$ can be reliably received. We measure channel quality for channel i using Shannon's mutual information $I(x_i; y_i)$. Due to the strict delay constraint relative to channel variations, we treat the mutual informations as random variables with distributions determined by the channel model.

For a single description system, the source codeword, $\hat{s}$, can be reliably decoded only if the *total* channel quality is high enough to support the transmission rate. One the other hand, for a multiple description system, each source codeword $\hat{s}_i$ can be decoded if the quality of the corresponding *individual* channel is high enough. Specifically, in terms of the mutual informations, $\hat{s}$ can be successfully decoded in a single description system when $I(x_1; y_1) + I(x_2; y_2) > R_{\rm sd}$ and $\hat{s}_i$ can be successfully decoded in a multiple description system when $I(x_i; y_i) > R_{{\rm md},i}$.

Fig. 3 compares the two systems when the multiple description system is designed to achieve the same distortion as a single description system if all source codewords are successfully decoded (*i.e.*, in region III). Furthermore, in region I, both systems fail to decode and again have the same distortion. In regions II and V the single description system is superior since the channel conditions are such that zero or one source codeword is decoded in the

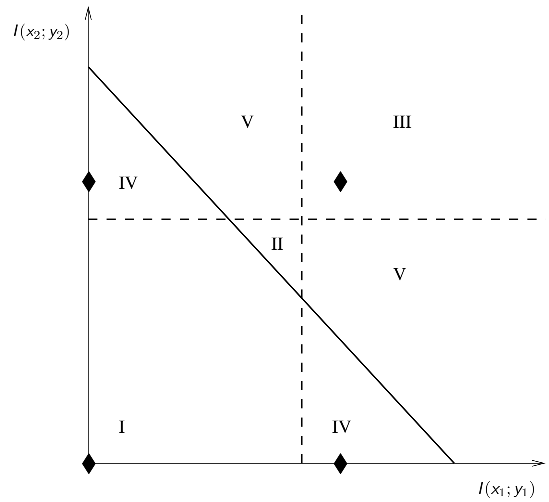

Fig. 3. Diagram of successful decoding regions for single and multiple description systems designed to have the same distortion when all codewords are received. The region above the solid line represents channel conditions where the total channel mutual information is greater than the source coding rate for the single description system and thus the single description codeword is received. The regions above and to the right of the vertical and horizontal dashed lines represent channel conditions where the channel mutual information exceeds the source coding rate for the multiple description source codewords $\hat{s}_1$ and $\hat{s}_2$ respectively resulting in successful decoding of $\hat{s}_1$ and $\hat{s}_2$.

multiple description system, while the single description source codeword is reliably received. Conversely, in region IV the multiple description system is superior since one source codeword is received while the single description system fails to decode.

Intuitively, we expect the shape of the probability distributions of $I(x_i; y_i)$, $i = 1, 2$ to influence which of the two systems offers better performance. If regions II and V are more probable, the single description system will be superior; on the other hand, if regions IV are more likely, the multiple description system will be superior.

As a specific example, in the classic multiple description problem modeling link failure or packet erasure [2], each channel is either off, in which case no information can be communicated, or supports a particular rate R. The four channel conditions for this scenario are indicated by $\blacklozenge$'s in Fig. 3 for an example packet erasure channel. For such discrete models, multiple description coding is clearly superior since both the single and multiple description systems achieve the same distortions in regions I and III, but the single description system fails completely in region IV. In this region, the multiple description system recovers one source codeword and produces a low fidelity reconstruction of the source.

In the sequel, we study the important case of a white Gaussian source transmitted over parallel independent Rayleigh fading channels corrupted by additive white

Gaussian noise. Specifically we show that at high signal-to-noise ratios, obtaining diversity at the physical layer via parallel channel coding in a single description system is superior to obtaining diversity at the application layer via multiple description source coding in a multiple description system. However, the multiple description system is superior to single description systems with repetition coding.

II. System Model

For simplicity, we model the source as a zero-mean, unit-variance circularly symmetric complex Gaussian random process, $s(t)$, band-limited to S Hz. As a reasonable model for a system employing, *e.g.*, extra bandwidth to exploit diversity in two sub-bands, the channels we consider consist of ideal band-limited filters with bandwidth W Hz and attenuation a_i corrupted by additive white Gaussian noise (AWGN). We denote the ratio of channel bandwidth to source bandwidth (*i.e.*, processing gain) as $L = W/S$.

A baseband equivalent, discrete-time model for the channel has

$$y_1[n] = a_1\, x_1[n] + z_1[n] \quad (1)$$

$$y_2[n] = a_2\, x_2[n] + z_1[n]\ , \quad (2)$$

where a_i captures the effects slow, frequency non-selective multipath fading in the respective sub-band, and z_i captures the effects of additive noise and other interference in the system. Statistically, we model a_i as complex Gaussian random variables, and $z_i[n]$ as complex Gaussian random sequences, all being zero-mean, mutually independent, and circularly-symmetric. This implies, for example, that $|a_i|$ is Rayleigh distributed, and $|a_i|^2$ is exponentially distributed. Without loss of generality, we normalize $\mathrm{E}\left[|a_i|^2\right] = \mathrm{E}\left[|z_i[n]|^2\right] = 1$, and $\mathrm{E}\left[|x_i[n]|^2\right] = \mathrm{SNR}/2$, where SNR is the signal-to-noise ratio (SNR), a key parameter of the channel model.

A. Source Coding Schemes

We consider two possible source coding techniques: single description coding and multiple description coding. In both cases distortion is measured according to mean-square error.

1) Single Description Source Coding: As is well-known, the rate (in nats/channel sample) required for single description source coding is given by the rate-distortion function of a Gaussian source [3]

$$R_{\mathrm{sd}}(D) = \frac{1}{L}\log\frac{1}{D}. \quad (3)$$

The rate (3) can be achieved using long Gaussian random codebooks achieving distortion D.

2) Multiple Description Coding: In multiple description coding, the source is represented by two (or more) descriptions such that each description alone provides a low fidelity reconstruction of the source while combining descriptions provides a high fidelity representation. Diversity can then be achieved at the application layer, by sending the separate descriptions over independent channels.

The rates and distortions achievable by coding a complex Gaussian source into two equal-rate descriptions with a total rate of R_{md} nats per channel sample, (*i.e.*, each description requires $R_{\mathrm{md}}/2$ nats) satisfy [2]

$$R_{\mathrm{md}}(D_0, D_1) = \frac{1}{L}\log\frac{1}{D_0} + \frac{1}{L}\log\frac{(1-D_0)^2}{(1-D_0)^2 - (1-2D_1+D_0)^2}\ , \quad (4)$$

where D_0 is the distortion when both descriptions are received and D_1 is the description when only a single description is received.

B. Channel Coding Schemes

There are a variety of approaches to channel coding in the context of the systems in Figs. 1 and 2. We focus on parallel channel coding for the single description system in Fig. 1 and independent channel coding over the two channels in the multiple description system in Fig. 2. To examine fundamental performance and compare between systems, we analyze random coding over non-ergodic channels using outage probability [7] as a performance measure. Briefly, because the mutual information I, corresponding to the supportable transmission rate of the channel, is a function of the fading coefficients, it too is a random variable. For fixed transmission rate R (in nats/channel use), the outage probability $\Pr[I < R]$ measures channel coding robustness to uncertainty in the fading coefficients.

We now summarize outage probability performance for parallel channel coding and coding independently over the two channels.

1) Parallel Channel Coding: Using a pair of jointly-designed complex Gaussian random codebooks, the mutual information for the parallel channels as a function of the fading coefficients is

$$I_{\mathrm{pc}}(\mathrm{SNR}) = \log\left(1 + (\mathrm{SNR}/2)|a_1|^2\right) + \log\left(1 + (\mathrm{SNR}/2)|a_2|^2\right)\ ,$$

where the factor of a half in the SNR results from spreading power over twice the bandwidth. In this case, the outage probability is

$$p_{\rm pc}^{\rm out}({\rm SNR},R) \stackrel{\triangle}{=} \Pr[I_{\rm pc}({\rm SNR}) < R]$$
$$= \Pr\left[|a_1|^2 + |a_2|^2 + ({\rm SNR}/2)|a_1|^2|a_2|^2 < \frac{e^R-1}{{\rm SNR}/2}\right] . \quad (5)$$

A simpler, but less powerful, parallel channel code, is a repetition code across the two channels using a common Gaussian random codebook $x_1 = x_2$. The mutual information for a repetition coding approach is given by

$$I_{\rm rc}({\rm SNR}) = \log\left(1 + ({\rm SNR}/2)(|a_1|^2 + |a_2|^2)\right) ,$$

so that the outage probability is

$$p_{\rm rc}^{\rm out}({\rm SNR},R) \stackrel{\triangle}{=} \Pr[I_{\rm rc}({\rm SNR}) < R]$$
$$= \Pr\left[|a_1|^2 + |a_2|^2 < \frac{e^R-1}{{\rm SNR}/2}\right] . \quad (6)$$

For Rayleigh fading, (6) becomes

$$p_{\rm rc}^{\rm out}({\rm SNR},R) = 1 - \left(1 + \frac{e^R-1}{{\rm SNR}/2}\right)\exp\left(-\frac{e^R-1}{{\rm SNR}/2}\right) . \quad (7)$$

2) Independent Channel Coding: To support multiple description source coding, we employ independent channel codes on each of the channels (1)-(2). The mutual information of either channel using Gaussian random codebooks is given by

$$I_{\rm ic}({\rm SNR}) = \log(1 + ({\rm SNR}/2)|a|^2) ,$$

so that the outage probability of each one of the channels is

$$p_{\rm ic}^{\rm out}({\rm SNR},R) \stackrel{\triangle}{=} \Pr[I_{\rm ic}({\rm SNR}) < R]$$
$$= \Pr\left[|a|^2 < \frac{e^R-1}{{\rm SNR}/2}\right] . \quad (8)$$

Note that because of the independence of the channels, the probability that both the channels experience outage is $[p_{\rm ic}^{\rm out}({\rm SNR},R)]^2$. For Rayleigh fading, (8) becomes

$$p_{\rm ic}^{\rm out}({\rm SNR},R) = 1 - \exp\left(-\frac{e^R-1}{{\rm SNR}/2}\right) \quad (9)$$

C. Average Distortion Performance

To compare schemes, we compute statistics on the end-to-end distortion of the various systems. For simplicity of exposition, we only treat the mean-distortion computations. We also optimize over target distortion levels for each scheme, since a general relationship between single and multiple description distortions is not presently known.

D. Single Description System

The performance of single description systems depends upon a single outage event. Choosing a target distortion D requires a channel coding rate of $R_{\rm sd}(D)$ given by (3). When an outage occurs (with probability $p^{\rm out}({\rm SNR}, R_{\rm sd}(D))$), the source incurs distortion 1 (more generally σ_s^2); on the other hand, when no outage occurs (with probability $1 - p^{\rm out}({\rm SNR}, R_{\rm sd}(D))$), the source incurs distortion D.

In the case of parallel channel coding, the average distortion performance is given by

$$D_{\rm sd,pc}({\rm SNR}) = \min_D \left\{(1 - p_{\rm pc}^{\rm out}({\rm SNR}, R_{\rm sd}(D))) \cdot D + p_{\rm pc}^{\rm out}({\rm SNR}, R_{\rm sd}(D)) \cdot 1\right\} , \quad (10)$$

with $R_{\rm sd}(D)$ given by (3) and $p_{\rm pc}^{\rm out}({\rm SNR},R)$ given by (5)

For the special case of repetition coding across the parallel channels, the average distortion performance is given by

$$D_{\rm sd,rc}({\rm SNR}) = \min_D \left\{(1 - p_{\rm rc}^{\rm out}({\rm SNR}, R_{\rm sd}(D))) \cdot D + p_{\rm rc}^{\rm out}({\rm SNR}, R_{\rm sd}(D)) \cdot 1\right\} , \quad (11)$$

with $R_{\rm sd}(D)$ given by (3) and $p_{\rm rc}^{\rm out}({\rm SNR},R)$ given by (6).

For single description systems the target distortion D must be chosen to balance two competing trends. Small D (large R) reduces the distortion when there is no outage, but also increase the outage probability. Large D (small R) reduces the outage probability but increases the distortion when there is an outage. Balancing these competing effects is the objective of the optimizations in (10) and (11).

E. Multiple Description System

In contrast with single description systems, the performance for multiple description systems depends upon two outage events. Choosing a pair of target distortions D_0 and D_1 requires a channel coding rate of $R_{\rm md}(D_0,D_1)/2$ given by (4) for each channel. If an outage occurs on both channels (with probability $[p_{\rm ic}^{\rm out}({\rm SNR}, R_{\rm md}(D_0,D_1)/2)]^2$), the source incurs distortion 1 (more generally σ_s^2). On the other hand, when an outage occurs on only one channel, which occurs with probability

$$2 \cdot p_{\rm ic}^{\rm out}({\rm SNR}, R_{\rm md}(D_0,D_1)/2) \cdot [1 - p_{\rm ic}^{\rm out}({\rm SNR}, R_{\rm md}(D_0,D_1)/2)] , \quad (12)$$

the successfully received codeword provides a low fidelity reconstruction of the source incurring distortion D_1. When no outage occurs (with probability

$[1 - p_{\rm ic}^{\rm out}(\mathrm{SNR}, R_{\rm md}(D_0, D_1))]^2$), both received codewords are combined to form a high fidelity reconstruction incurring distortion D_0.

Thus with independent channel coding, the average distortion performance is given by

$$D_{\rm md,ic}(\mathrm{SNR}) = \min_{D_0, D_1} \Big\{ \left[p_{\rm ic}^{\rm out}(\mathrm{SNR}, R_{\rm md}(D_0, D_1)/2)\right]^2 + 2 \cdot D_1 \cdot p_{\rm ic}^{\rm out}(\mathrm{SNR}, R_{\rm md}(D_0, D_1)/2) \cdot \left[1 - p_{\rm ic}^{\rm out}(\mathrm{SNR}, R_{\rm md}(D_0, D_1)/2)\right] + \left[1 - p_{\rm ic}^{\rm out}(\mathrm{SNR}, R_{\rm md}(D_0, D_1)/2)\right]^2 \cdot D_0 \Big\} , \quad (13)$$

with $R_{\rm md}(D_0, D_1)$ given by (4) and $p_{\rm ic}^{\rm out}(\mathrm{SNR}, R)$ given by (8).

III. Asymptotic Analysis

In order to identify the benefits of various diversity schemes we consider the limiting behavior of the average distortion as a function of SNR as $\mathrm{SNR} \to \infty$. Since the distortion for our systems of interest has the form $D(\mathrm{SNR}) \propto \mathrm{SNR}^{-\Delta}$ for some Δ at high SNR, we define the average distortion exponent as

$$\Delta = \lim_{\mathrm{SNR}\to\infty} \frac{-\log D(\mathrm{SNR})}{\log \mathrm{SNR}} . \quad (14)$$

We omit a detailed analysis and present our results for the average distortion exponents of various systems in Table I. We compare performance of single and multiple description diversity systems to performance of a single AWGN channel with fading (*i.e.*, no diversity), and a parallel AWGN channel (*i.e.*, infinite diversity).

TABLE I

Average distortion exponents.

Channel	Source Code	Channel Code	Δ
Parallel AWGN	SD	PC	$2L$
Parallel Rayleigh	SD	PC	$2L/(L+1)$
Parallel Rayleigh	MD	IC	$4L/(2L+3)$
Parallel Rayleigh	SD	RC	$2L/(L+2)$
Single Rayleigh	SD	IC	$L/(L+1)$

Fig. 4 shows the average distortion for various systems where the parameters in the optimizations (10), (11), and (13) have been numerically computed for the case $L = 1$. As the plot indicates, the difference in performance suggested by the asymptotic results in Table I becomes evident even at reasonable SNR.

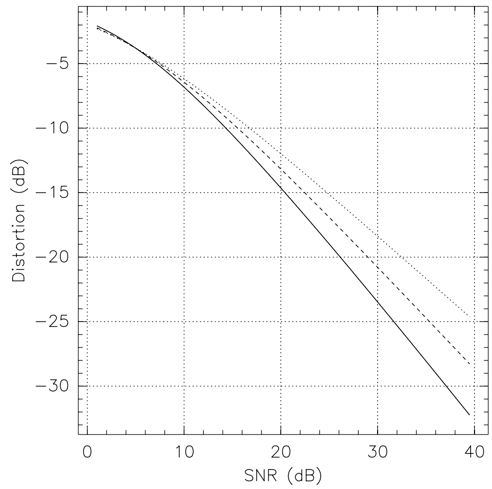

Fig. 4. The lines represent the average distortion performance of single description source coding with repetition coding (top dotted line), multiple description source coding with independent channel coding (middle dashed line), and single description source coding with parallel channel coding (bottom solid line) with a processing gain of 1 (*i.e.*, $L = 1$).

IV. Concluding Remarks

We showed that single description source coding combined with parallel channel coding achieves a lower expected distortion than multiple description source coding with independent channel coding when transmitting a Gaussian source over independent Rayleigh fading channels. For large processing gains, however, the performance gap is small and multiple description systems may be more desirable in practice due to other issues such as complexity, ease of deployment, or when other channel effects (*e.g.*, congestion) are a concern. Topics for future work include investigating whether qualitatively similar results hold for sources with memory and for practical systems as opposed to information theoretically optimal systems for i.i.d. sources.

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