

SIGNAL PROCESSING TECHNIQUES FOR EFFICIENT USE OF TRANSMIT DIVERSITY IN WIRELESS COMMUNICATIONS

Gregory W. Wornell

Mitchell D. Trott

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

A class of efficient strategies for exploiting transmit antenna diversity on fading channels is developed. These techniques, which we refer to as linear antenna precoding, asymptotically transform a nonselective Rayleigh fading channel into a nonfading, simple white marginally Gaussian noise channel with no intersymbol interference. Linear antenna precoding requires no additional power or bandwidth, and is also attractive in terms of computation, robustness and delay considerations.

1. INTRODUCTION

In wireless applications, fading due to multipath propagation severely impacts system performance. However, the effects of fading can be substantially mitigated through the use of diversity techniques in such systems via appropriately designed signal processing algorithms at both the transmitters and receivers.

We focus on the use of spatial diversity in nonselective fading environments, a situation that applies when time variations in the channel are very slow relative to the symbol duration, and when frequency variations are on scales much larger than the system bandwidth. Spatial diversity involves the use of multiple antennas at the receiver and/or the transmitter. We focus on the latter case, which is termed transmit diversity. This form of diversity is particularly attractive in applications such as broadcasting and forward-link (base-to-mobile) transmission in cellular systems.

For such scenarios, we develop a class of efficient and practical linear signal processing algorithms for exploiting transmit diversity on nonselective fading channels without incurring bandwidth expansion. Moreover, these algorithms can be efficiently combined with other forms of diversity and error-correction coding to further improve system performance. Our framework is also convenient for analyzing and relating several novel strategies proposed in the literature, including, e.g., [1] [2] [3] [4].

2. LINEAR ANTENNA PRECODING

Consider the equivalent discrete-time baseband model of a passband channel for a system with M transmit antennas, where the (generally complex-valued) transmission from the m th antenna we denote using $y_m[n]$ for $m = 0, 1, \dots, M-1$.

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At the receiver we obtain

$$r[n] = w[n] + \sum_{m=0}^{M-1} a_m y_m[n], \quad (1)$$

where $w[n]$ denotes the complex-valued, zero-mean, white Gaussian noise receiver noise, which has variance \mathcal{N}_0 and is independent of the fading coefficients a_0, a_1, \dots, a_{M-1} . Given sufficient physical separation among the constituent antennas, the fading coefficients can be modeled as mutually independent, complex-valued, zero-mean Gaussian random variables with variance σ_a^2 . We assume that these coefficients are not known at the transmitter (i.e., feedback is not viable), but *are* known (or, more typically, can be reliably estimated) at the receiver.

For this channel, we consider a transmitter structure in which the bit stream is first processed by a single, suitably-designed error-correcting coder. The resulting coded symbol stream $x[n]$ is then processed by a linear processor at each of the constituent antennas of the transmitter. We refer to this second stage of processing as "linear antenna precoding." At the receiver, the observations $r[n]$ are first processed by a linear equalizer to generate the equalized signal $\hat{x}[n]$, which is subsequently decoded.

3. LTI ANTENNA PRECODING

When the linear antenna precoding takes the form of linear time-invariant (LTI) filtering, the transmission from the m th antenna is

$$y_m[n] = \frac{1}{\sqrt{M}} \sum_{k=-\infty}^{+\infty} h_m[n] x[n-k], \quad (2)$$

where we refer to the (generally complex-valued) unit-sample response $h_m[n]$ as a "signature" of the precoder. The associated Fourier transform of each signature will be denoted by $H_m(\omega)$. We refer to the set of M signatures as the signature set, and to a collection of signature sets $\{h_m[n]\}_{m=0}^{M-1}$ indexed by the array size M as a *family* of signature sets.

We restrict our attention to families of signature sets meeting the following conditions.

Definition 1 *A family of signature sets is termed admissible if the following conditions are satisfied:*

$$\frac{1}{M} \sum_{m=0}^{M-1} |H_m(\omega)|^2 = 1, \quad \text{all } \omega \quad (3a)$$

$$\lim_{M \rightarrow \infty} \frac{1}{M} \sum_{m=0}^{M-1} H_m(\omega) H_m^*(\nu) = 0, \quad \omega \neq \nu. \quad (3b)$$

Condition (3a) ensures that the total average transmitted power is independent of M , while (3b) ensures that certain attractive asymptotic characteristics can be achieved.

An example of a linear antenna precoding system meeting the conditions of Definition 1 is one using antenna signatures with ideal bandpass characteristics, i.e.,

$$H_m(\omega) = \begin{cases} \sqrt{M} & m\pi/M < |\omega| < (m+1)\pi/M \\ 0 & \text{elsewhere in } |\omega| < \pi. \end{cases}$$

This corresponds to the notion of assigning each antenna a distinct portion of the available bandwidth.

Although ideal bandpass signatures are, of course, unrealizable, families of practical finite-length signatures can be readily constructed. We consider signatures of length M , for which we can construct the matrix representation

$$\mathbf{H} = \begin{bmatrix} h_0[0] & h_0[1] & \cdots & h_0[M-1] \\ h_1[0] & h_1[1] & \cdots & h_1[M-1] \\ \vdots & \vdots & \ddots & \vdots \\ h_{M-1}[0] & h_{M-1}[1] & \cdots & h_{M-1}[M-1] \end{bmatrix}. \quad (4)$$

For such signatures, we have the following theorem [5].

Theorem 1 *For a family of signature sets whose constituent signatures have length M to be admissible in the sense of Definition 1, it is sufficient that \mathbf{H} in (4) be a unitary matrix, i.e., that each signature set consist of orthogonal signatures.*

Theorem 1 leads to a broad class of systems. For example, when we choose $\mathbf{H} = \mathbf{I}$ where \mathbf{I} is the $M \times M$ identity matrix so that $h_m[n] = \delta[n-m]$, we obtain a scheme explored both by Wittneben [1] for the case $M = 2$ and, more generally, by Winters [4]. We can also choose $\mathbf{H} = \mathbf{F}$ or $\mathbf{H} = \mathbf{\Xi}$, which are the discrete Fourier transform (DFT) and (normalized) Hadamard matrices, respectively. For $M = 2$, both are equivalent to a scheme also explored by Wittneben [1].

3.1. System Characteristics and Receiver Design

An important interpretation of linear antenna precoding is that it effectively transforms the original nonselective fading channel into a frequency-selective fading channel. To see this, we observe, combining (1) with (2), that the received signal can be expressed in the form

$$r[n] = a[n] * x[n] + w[n] \quad (5)$$

where

$$a[n] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} a_m h_m[n] \quad (6)$$

is the impulse response of the “effective” channel generated by the antenna precoder. This channel has frequency response

$$A(\omega) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} a_m H_m(\omega), \quad (7)$$

which is a zero-mean, 2π -periodic, Gaussian random process in frequency ω , with variance σ_a^2 .

We restrict our attention to linear equalizers for exploiting the inherent frequency diversity, which remain practical even when the number of antennas is large, so that our receiver structure consists of an equalizer followed by a decoder. Specifically, for an LTI equalizer with unit-sample response $b[n]$, the equalized signal is

$$\hat{x}[n] = b[n] * r[n] = \sum_k b[k] r[n-k]. \quad (8)$$

We consider equalizers whose frequency response takes the form $B(\omega) = f(A(\omega))$ where $A(\omega)$ is as defined in (7), and where $f(\cdot)$ is a complex-valued function that satisfies some mild admissibility conditions [5].

Combining (8) with (5) we obtain

$$\hat{x}[n] = c[n] * x[n] + b[n] * w[n], \quad (9)$$

where $c[n] = a[n] * b[n]$, so that $C(\omega) = A(\omega)B(\omega)$ is the effective frequency response after equalization. For the class of equalizers under consideration, we have that $\mu_b = E[B(\omega)]$, $\mu_c = E[C(\omega)]$, $\sigma_b^2 = \text{var} B(\omega)$, and $\sigma_c^2 = \text{var} C(\omega)$.

In turn, we obtain the following theorem that asymptotically characterizes the composite system consisting of the antenna precoder, channel, and equalizer [5].

Theorem 2 *Let $x[n]$ be a sequence of zero-mean uncorrelated symbols, each with energy \mathcal{E}_s . Furthermore, for every M , let a_m for $m = 0, 1, \dots, M-1$ be a collection of independent complex-valued Gaussian random variables, each with mean zero and variance σ_a^2 . Finally, suppose $b[n]$ is the unit-sample response of an admissible equalizer, and that the antenna precoder signature sequences $h_m[n]$ satisfy the conditions (3). Then, as $M \rightarrow \infty$, we have, for each n ,*

$$\hat{x}[n] \xrightarrow{\text{m.s.}} \mu_c x[n] + v[n], \quad (10)$$

where $v[n]$ is a complex-valued, marginally Gaussian, zero-mean white noise sequence, uncorrelated with the input symbol stream $x[n]$ and having variance

$$\text{var} v[n] = \mathcal{E}_s \sigma_c^2 + \mathcal{N}_0 (\sigma_b^2 + |\mu_b|^2). \quad (11)$$

Theorem 2 asserts that given transmit antenna diversity of this form, the channel “seen” by the coded symbol stream is transformed from a fading channel into a marginally Gaussian white noise channel. From (11), we see there are two components in this equivalent noise: one component is due to the original receiver noise, the second is due to intersymbol interference (ISI) that is generated by the transmit diversity, and hence has a variance that scales with the symbol energy. In effect, we see that this ISI is transformed into a comparatively more benign form of uncorrelated, additive noise. This theorem is the transmit diversity counterpart of one developed for a class of linear temporal diversity schemes in [6] [7], and the salient features of the characterization are analogous.

As Theorem 2 suggests, in the absence of additional coding, good performance is achieved using simple symbol-by-symbol detection after the equalizer. Such simplifications mean that it is practical to combine this antenna diversity with additional error-correction coding for the Gaussian channel. For example, if trellis-coding is used, Viterbi decoding can be used after equalization as if the channel were Gaussian.

System performance depends strongly on the signal-to-noise ratio (SNR) associated with the equivalent channel, which in turn depends strongly on the choice of equalizer. A useful criterion for equalizer design is to select among admissible equalizers that yielding the largest SNR in the equivalent channel, which leads to [5]

$$B(\omega) \propto \frac{A^*(\omega)}{|A(\omega)|^2 + \xi_0}, \quad (12)$$

where $\xi_0 = \mathcal{N}_0/\mathcal{E}_s$. Note that, coincidentally, this is a minimum mean-square error type equalizer, which is attractive from the point of view of implementation. Also, note that the numerator of (12) is a conventional matched filter (i.e., RAKE receiver [8]), so that the denominator can be viewed as an additional compensation stage.

3.2. Performance

The asymptotic performance characteristics ($M \rightarrow \infty$) of the optimized system also follow from the equivalent quasi-Gaussian channel. Specifically, while the SNR in the original channel, i.e.,

$$\alpha_0(\omega) = \frac{|A(\omega)|^2 \mathcal{E}_s}{N_0} \quad (13)$$

is both random and varies as a function of frequency ω , with the optimum equalizer, i.e., (12), we immediately obtain that the SNR in the equivalent channel is a deterministic constant of the form

$$\gamma_0 = \frac{1}{E\left[\frac{1}{\alpha_0+1}\right]} - 1 = \frac{1}{\zeta_0 e^{\zeta_0} E_1(\zeta_0)} - 1, \quad (14)$$

where $1/\zeta_0 = E[\alpha_0(\omega)] = \sigma_a^2 \mathcal{E}_s / N_0$ is the average SNR in the original channel, and where $E_1(\cdot)$ denotes the exponential integral defined via

$$E_1(\nu) = \int_{\nu}^{\infty} \frac{e^{-t}}{t} dt. \quad (15)$$

A useful notion of capacity for our transmit diversity scheme results when we ignore the higher-order statistical dependencies in the equivalent channel of Theorem 2 and view the channel as strictly Gaussian channel. Specifically,

$$C = \log(1 + \gamma_0) = -\log(\zeta_0 e^{\zeta_0} E_1(\zeta_0)) \quad (16)$$

suggests the achievable bit rate when sufficient coding is used prior to precoding.

This effective capacity can be compared to that of related transmit diversity systems without such stringent computational constraints [9]. For example, when we remove the constraint that the front end of our receiver be a linear equalizer and allow an arbitrarily complex decoder, the asymptotic capacity increases to

$$C_L = E[\log(1 + \alpha_0)] = e^{\zeta_0} E_1(\zeta_0) \quad (17)$$

in the case $h_m[n] = \delta[n - m]$. When, in addition, we remove the constraint that the antenna precoding be linear and allow an arbitrarily complex encoder at each antenna, then the asymptotic capacity increases still further to the conventional Gaussian channel capacity

$$C_T = \log(1 + E[\alpha_0]) = \log(1 + 1/\zeta_0),$$

which, it should be pointed out, is also what can be achieved using unconstrained receive diversity [9].

Even with no additional coding, the use of antenna precoding leads to substantially reduced bit-error rates over systems without such transmit diversity. As an illustration, when $x[n]$ is an uncoded quadrature phase-shift keying (QPSK) symbol stream, the bit error probability given infinite transmit diversity is

$$\mathcal{P} = Q(\sqrt{\gamma_0}) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\gamma_0}}^{\infty} e^{-t^2/2} dt, \quad (18)$$

where γ_0 is as defined in (14). For comparison, without transmit diversity the QPSK bit error probability is [8]

$$\mathcal{P}_0 = \frac{1}{2} \left(1 - \frac{1}{\sqrt{2\zeta_0 + 1}} \right). \quad (19)$$

The asymptotic bit error rate (18) provides a useful bound on what can be obtained in practice with finite transmit diversity ($M < \infty$). However, the LTI antenna precoding we have developed thus far is impractical because the optimum equalizer is unrealizable and approximations incur excessive delay. We now consider a generalization that allows such problems to be circumvented.

4. LPTV ANTENNA PRECODING

We now allow the processing at each antenna to take the form of more general linear periodically time-variant (LPTV) filtering, but restrict our attention to a particular class of such systems that admit the following factorization. The coded symbol stream $x[n]$ is first processed by a common LPTV prefilter that is time-varying with some period $K \geq 2$ and has length M , and whose kernel we denote by $g[n; k]$. The result,

$$y[n] = \sum_k g[n; k] x[n - k], \quad (20)$$

is then subsequently processed at each of the antennas. Specifically, this prefiltered stream is modulated at each antenna by a different M -periodic sequence, i.e.,

$$y_m[n] = \frac{1}{\sqrt{M}} \tilde{h}_m[n] y[n], \quad (21)$$

where $\tilde{h}_m[n]$ is the generally complex-valued periodic sequence associated with the m th antenna. We use $h_m[n]$ to denote a single period of this modulating sequence—i.e., $h_m[n] = \tilde{h}_m[n]$ for $0 \leq n \leq M - 1$ and is zero otherwise—and refer to this as the “signature” of the associated antenna.

4.1. Signature Design

Using (21) in (1), we obtain that the response of the channel to the prefiltered symbol stream $y[n]$ is

$$r[n] = w[n] + \tilde{a}[n] y[n], \quad (22)$$

where

$$\tilde{a}[n] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} a_m \tilde{h}_m[n] \quad (23)$$

is an M -periodic fading sequence.

From (22) we see that the signature modulation subsystem effectively transforms the original nonselective fading channel into a time-selective one. As such, this transformation is the dual of the nonselective to frequency-selective transformation we explored in Section 3. The maximum achievable time diversity benefit is obtained when the fading is independent among time-samples within a period, and thus we design our signature sequences to ensure that this condition is met. Again collecting our signatures into a matrix \mathbf{H} of the form (4), it is straightforward to verify [5] that the coefficients $a[0], a[1], \dots, a[M - 1]$ are statistically independent if and only if \mathbf{H} is a unitary matrix.

Evidently, there are infinitely many signature sets that ensure the independent fading condition. One example corresponds to $\mathbf{H} = \mathbf{I}$, but this symbol-dealing strategy has high peak-power requirements. An alternative is $\mathbf{H} = \mathbf{F}$, where \mathbf{F} is the suitably-sized DFT matrix, which can be interpreted as an efficient discrete-time variant of the phase-sweeping transmit diversity system explored by Hiroike et al. [2] and Weerackody [3]. Finally, the choice $\mathbf{H} = \mathbf{\Xi}$ where $\mathbf{\Xi}$ is the Hadamard matrix, is particularly attractive both from the point of view of peak-power and computational complexity.

4.2. Prefilter Design

We next turn our attention to the design of a LPTV prefilter that can exploit the time diversity generated by the signature modulation process. Although prefiltering is not strictly necessary if coding is used, [6] suggests that given computational constraints the best diversity benefit is often

achieved by combining (or sometimes even replacing) coding with suitably designed prefiltering. In particular, the bandwidth-preserving LPTV maximally-spread precoders developed in [7] (after [6]) are naturally suited as prefilters for our application. For time-selective fading channels, these binary-valued and finite impulse response orthogonal systems provide the optimum linear diversity benefit with very low computational complexity [7] [5].

4.3. System Characteristics and Receiver Design

For these systems, the natural linear equalizer consists of the cascade of two stages:

$$\hat{y}[n] = \tilde{b}[n] r[n], \quad \text{then} \quad \hat{x}[n] = \sum_k g[k; n] \hat{y}[n - k], \quad (24)$$

where $\tilde{b}[n]$ is a suitable equalizer for the time-selective fading, and where we recognize that the postfilter $g[k; n]$ corresponds to the inverse of the prefilter since the system is orthogonal. Hence,

$$\hat{y}[n] = \tilde{c}[n] y[n] + \tilde{b}[n] w[n], \quad (25)$$

where $\tilde{c}[n] = \tilde{a}[n] \tilde{b}[n]$. Using orthogonal signatures and equalizers of the form $\tilde{b}[n] = f(\tilde{a}[n])$, where $f(\cdot)$ again meets some mild admissibility conditions [5], we obtain $\text{var } \tilde{a}[n] = \sigma_a^2$ and, in turn, $\mu_b = E[\tilde{b}[n]]$, $\mu_c = E[\tilde{c}[n]]$, $\sigma_b^2 = \text{var } \tilde{b}[n]$, and $\sigma_c^2 = \text{var } \tilde{c}[n]$.

The counterpart of Theorem 3 for the case of LPTV precoding then follows immediately from the results of [6] [7].

Theorem 3 *Let $x[n]$ be a sequence of zero-mean uncorrelated symbols, each with energy \mathcal{E}_s . Furthermore, for every M , let a_m for $m = 0, 1, \dots, M - 1$ be a collection of independent complex-valued Gaussian random variables, each with mean zero and variance σ_a^2 . Finally, suppose $\tilde{b}[n]$ is an admissible equalizer, that the length- M antenna precoder signature sequences form an orthonormal set, and that maximally-spread prefilters of spread M are used. Then, as $M \rightarrow \infty$, we have, for each n ,*

$$\hat{x}[n] \xrightarrow{\text{m.s.}} \mu_c x[n] + v[n], \quad (26)$$

where $v[n]$ is a complex-valued, marginally Gaussian, zero-mean white noise sequence, uncorrelated with the input symbol stream $x[n]$ and having variance

$$\text{var } v[n] = \mathcal{E}_s \sigma_c^2 + \mathcal{N}_0 (\sigma_b^2 + |\mu_b|^2). \quad (27)$$

The SNR in the equivalent channel of Theorem 3 is, analogously, maximized when

$$\tilde{b}[n] \propto \frac{\tilde{a}^*[n]}{|\tilde{a}[n]|^2 + \xi_0}, \quad (28)$$

where, again, $\xi_0 = \mathcal{N}_0/\mathcal{E}_s$. As a result of the new equalizer structure, systems with optimized LPTV antenna precoding retain the same attractive asymptotic characteristics as those based on LTI systems, but system delay is substantially less.

4.4. Performance

Using maximally-spread prefilters and the optimum equalizers, the performance of LPTV antenna precoding is depicted in Fig. 1 for several different antenna array sizes M . Also, superimposed is the performance without antenna diversity, the performance with infinite transmit diversity $M \rightarrow \infty$.

These results, along with several others, are developed in detail in [5].

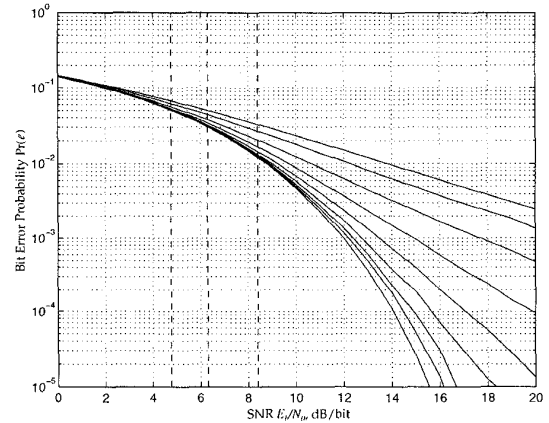


Figure 1. Bit error probabilities using uncoded QPSK on the Rayleigh fading channel with linear antenna precoding and linear equalization. The successively lower curves correspond to $M = 1, 2, 4, 8, 16, 32, 64, 128$, and $M \rightarrow \infty$ transmit antennas, respectively. From left to right, the dashed vertical lines denote the capacities C_T , C_L , and C , respectively.

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