Writing on Many Pieces of Dirty Paper at Once: The Binary Case

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Abstract — **We study the problem of sending a common message to several users on a channel with side information. Specifically, each user experiences an additive interference which is known only to the sender. The sender has to simultaneously adapt its transmitted signal to all the interferences. We derive upper and lower bounds for the special case of binary channels and derive some optimality conditions.**

I. Introduction

Study of the problem of channel coding with side information known only to the encoder originated with the work of Gel'fand and Pinsker [1], Heegard and El Gamal [2], and Costa [3]. The results and their extensions are increasingly finding wide ranging applications, from digital watermarking, to precoding for ISI channels, to coding for the MIMO broadcast channel, and to CEO problems. In this work consider a point-to-multipoint generalization of the problem, with an additive interference model. Specifically, we consider the scenario with one transmitter and K receivers, where all the receivers want a common message. There is no co-ordination among the receivers, and each receiver experiences an additive interference sequence that is known to the transmitter but not to any receiver. Since the rate of the common message is limited by the worst user, the transmitter has to deal with all the interference sequences simultaneously. In this paper, we restrict our attention to the binary case.

II. Binary Channels

Suppose the channel of user $k \in \{1, 2 \dots K\}$ is given by $Y_k = X \oplus S_k$, where X is the transmitted symbol and $\{S_k\}_{k=1}^{K}$ is the interference on the user channels, which is known non-causally to the transmitter but not to the receivers. We assume that X and S_k are each binary valued $\{0, 1\}$ and the addition is mod 2. Furthermore we assume that $\Pr(S_{k_1} \dots S_{k_m})$ does not depend on the specific choice of $k_1 \dots k_m \in \{1, 2 \dots K\}$. The transmitter wishes to send a message at the maximum possible rate R, so that all the receivers can reliably decode it. We present the following results:

Theorem 1 (Outer Bound) The maximum common rate for the K user binary channel satisfies:

$$R \leq 1 - \frac{1}{K} H(S_1 \oplus S_2, S_1 \oplus S_3 \dots S_1 \oplus S_K)$$

Theorem 2 (Inner Bound) An achievable rate R for the K user binary channels is

$$R = 1 - \left(1 - \frac{1}{K}\right) H(S_i \oplus S_j)$$

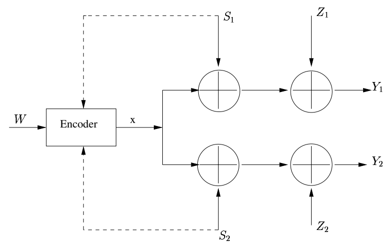

Figure 1: Binary Two User Channel

Proofs are developed in [4], where we also propose a novel coding scheme to achieve the inner bound. The basic idea is to divide a codeword into K sub-blocks and clean up the channel of a single user in each sub-block. The receivers use the knowledge of the reliability of different blocks in decoding. We also show that it is possible to improve the inner bound in Theorem 2 for $K \geq 3$, but the gains are not substantial. In general our inner and outer bounds are not equal. However they do agree in the following cases:

Lemma 1 (2 User Case) The capacity of 2 user binary case is given by $C = 1 - \frac{1}{2} H(S_1 \oplus S_2)$

Lemma 2 If $\{S_1, S_2 \dots S_K\}$ are mutually independent Bernoulli(1/2) random variables then the capacity is $C = \frac{1}{K}$ and is achieved through time-sharing.

Lemma 3 If $\{S_1, S_2 \dots S_K\}$ are mutually independent Bernoulli(q) random variables($q \leq 1/2$) then as $K \to \infty$ we have $C \to 1 - H(q)$ and this is achieved by ignoring the side information at the transmitter.

Note that if the side information was available at the receiver then the capacity would be $C = 1$. Our results show that there is a loss in capacity when there are $K \geq 2$ users. Simulations suggest the rather negative result that ignoring side information is quite close to the optimal rate over a large range of q even for a moderate number of users ($K \geq 5$); i.e., the side information generally can't be exploited much. For some achievable rates for the Gaussian case, see [4].

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